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Dimension 7 operators in the $b \to s$ transition.

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Abstract

We extend the low-energy effective field theory relevant for $b \to s$ transitions up to operators of mass-dimension 7 and compute the associated anomalous-dimension matrix. We then compare our findings to the known results for dimension 6 operators and derive a solution for the renormalization group equations involving operators of dimension 7. We finally apply our analysis to a particularly simple case where the Standard Model is extended by an electroweak-magnetic operator and consider limits on this scenario from the decays $B_s \to \mu^+\mu^-$ and $B \to K\nu\bar{\nu}$.

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1 Introduction

Flavour-violating processes are well-known as a central test of the Standard Model (SM). The pattern conceded to flavour transitions is indeed particularly constrained in this model, the Cabibbo-Kobayashi-Maskawa (CKM) matrix encoding the only source of flavour-breaking effects while only the charged currents of the weak interaction convey flavour-violation at tree-level. Consequently, flavour transitions also define closely watched observables in the quest for new physics and, due to the absence of positive deviations from the SM predictions, set serious constraints on the forms that physics beyond the SM (BSM) could take. One of the latest results is the observation by the LHCb collaboration of the decay $B_s^0 \to \mu^+\mu^-$ [1], with a branching ratio very compatible with the SM expectations (refer to [2] for a recent summary):

$$BR(B_s^0 \to \mu^+\mu^-)^{\text{exp.}} = (3.23 \pm 0.27) \cdot 10^{-9} \quad ; \quad BR(B_s^0 \to \mu^+\mu^-)^{\text{SM}} = (3.56 \pm 0.18) \cdot 10^{-9} \tag{1}$$

On the theoretical side, dedicated tools have been devised in order to study flavour-violating processes, in the form of low-energy effective field theories (EFT; refer to e.g. [3] for a review). These EFT's allow for a separation of the long distance, low-energy strong interaction effects and the short-distance, flavour-changing physics. In the context of B-physics, at the level of terms of dimension 4 and smaller, the relevant EFT consists in a QED×QCD model with five quark-flavours (u, d, c, s, b) and 3 charged leptons (e, μ, τ), as well as neutrinos. Then, in the low-energy processes involving those fields, at a scale $\mu_b \sim M_B$, the impact of higher-energy (top/Electroweak/Higgs/BSM; we assume here that there are no further low-energy 'invisible' fields) physics can be essentially encoded within operators of dimension > 4, collectively defining the 'effective hamiltonian'. The strong-interaction problem then consists in evaluating the S-matrix elements driven by these operators among physical (mesonic/baryonic) states: this question is answered, either through lattice-QCD, QCD sum rules, heavy-quark expansions and other theoretical descriptions of the non-perturbative strong-interaction effects, or phenomenologically, through an identification of the decay-constants by comparison with a few standard channels. The short-distance problem is summarized within the couplings multiplying the operators. Those must be matched at high-energy with the predictions of the 'more fundamental' theory (Standard Model, supersymmetry-inspired models, etc.): scattering amplitudes in both the EFT and the 'full-theory' are equated at the matching scale $\mu_0 \gtrsim M_W$, hence defining a boundary condition for the parameters of the EFT in terms of those of the underlying model.
To relate these two scales, μ_b and μ_0, one relies on the Renormalization Group Equations (RGE) driven by the renormalization of the EFT: leading logarithmic contributions can thus be consistently resummed. It is mostly in this part of the procedure that we will be interested in the following.
To fix notations, let us write the lagrangian density of the EFT under consideration:

$$\mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} \; + \; \imath \bar{f} \left[\gamma^\mu D_\mu - m_f \right] f \; - \; \mathcal{H}_{eff}^{(\text{dim}\geq 5)} \tag{2}$$

with $f = u, d, c, s, b, e, \mu, \tau, \nu_{e,\mu,\tau}$, $D_\mu = \partial_\mu - \imath e Q_f A_\mu - \imath g_S G^a_\mu$ the covariant derivative, m_f the mass of the fermion f, Q_f, its charge; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $G_{\mu\nu} = G^a_{\mu\nu} T^a = (\partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_S f^{abc} G^b_\mu G^c_\nu) T^a$ represent the electromagnetic and gluonic field tensors, respectively; T^a and f^{abc} denote the $SU(3)_c$ generators and structure constants; e and g_S respectively stand for the elementary electric charge and the strong coupling constant; $\alpha \equiv \frac{e^2}{4\pi}$, $\alpha_S \equiv \frac{g_S^2}{4\pi}$; $\mathcal{H}_{eff}^{(\text{dim}\geq 5)}$ is the effective hamiltonian.
The question arising at this point is that of the operators that one needs to consider in order to describe $b \to s$ transitions. Sensibly, people have considered, up to now, only operators of the lowest possible mass-dimension, that is dimension-6 operators[3]. This approach is justified by the suppression factor of $\frac{m_b}{M_Z} \sim 5 \cdot 10^{-2}$ which is expected for higher-dimensional operators. Moreover, it was (with reason) regarded as sufficient to confine to the smallest subset of dimension-6 operators that would close under renormalization and for which the classical models (SM, supersymmetry-inspired, etc.) would generate a non-trivial contribution:

$$\mathcal{H}_{eff}^{(\text{dim}=6)} = \sum_i C_i(\mu) O_i^{(\text{dim}=6)}(\mu) \tag{3}$$

where the list of operators O_i can be read in e.g. [3], [4] or [5] (with small variations; note that a factor $G_F/\sqrt{2}$ is conventionally factored out in the usual notations). The determination at leading order of the anomalous-dimension matrix for the four-quark operators of this subset is quite old: [6–10]. The renormalization of the magnetic and chromomagnetic operators can be found in [11], while [12] included the mixing of those with the

[3] In fact, the magnetic and chromo-magnetic operators have mass-dimension 5. However, they always appear, e.g. in the SM, with an additional mass-suppression, lowering their scale to an apparent dimension 6.

four-quark operators (a two-loop effect). One may refer to [13–15] for the analysis of the semi-leptonic operators. Later works have focussed on next-to-leading order ($O(\alpha_S)$) [5,16–20], electroweak ($O(\alpha)$; see e.g. [21]) and finally next-to-next-to-leading QCD ($O(\alpha_S^2)$; refer e.g. to the summary in [22]) effects: this formidable amout of work allows for a theoretical prediction, e.g. in the SM $\bar{B} \to X_s \gamma$ decay, competitive with experimental bounds.
In this paper we choose to adopt a different, more unprejudiced if somewhat more anecdotical, approach to the renormalization of the $b \to s$ EFT: we shall consider all possible (on-shell) operators up to mass-dimension 7 and compute the corresponding anomalous-dimension matrix at one-loop QCD order. As far as we know, no attempt has ever been made in that direction, due to the suppression of the order $\frac{m_b}{M_Z} \sim 5 \cdot 10^{-2}$ which one expects for higher-dimensional operators. In fact, if one considers the matching conditions in the particular case of the SM, with its restricted flavour-changing currents, the suppression would be even larger. The inclusion of higher-dimension operators hence admitedly appears in this concrete case more as a curiosity than a compelling necessity. There are however several reasons why analysing dimension 7 effects may not be completely irrelevant:

1. From the point of view of precision physics, one observes that $\alpha_S(M_Z) \sim \frac{m_b}{M_Z}$: with increasing precision in the SM evaluation as well as experimental measurements, observables shall eventually become sensitive to dimension 7 effects.

2. Certain new-physics contributions are actually 'hidden' dimension 7 effects; a simple example lies in the famous Higgs-penguin contributions to dimension 6 $bsll$-operators [23], relevant e.g. in supersymmetric models at large $\tan\beta$: the corresponding coefficients actually contain a factor m_b, formally increasing their order to dimension 7. Note however that other dimension 7 effects are not expected to receive an equally large $\tan\beta$-enhancement, although they should be relevant already at subleading order [24].

3. New-physics in dimension 6 operators is already stringently constrained, essentially enforcing the usual 'Minimal Flavour Violation' condition. A possible strategy to account for this absence of new-physics in flavour observables would be to reject it on operators of higher dimension. Similar proposals have been made in the neutrino sector to concile neutrino masses, baryon/lepton-number and lepton-flavour violating processes [25]. Note that, here, we do not propose a mechanism that would ensure the suppression of new physics by rejecting it on operators of dimension ≥ 7 (although this might be achievable, e.g. by assigning adequate charges under a discrete symmetry), but we simply mention this possibility as a motivation to consider such operators.

4. If one parametrizes new physics blindly in an expansion of SM dimension 6 operators [26,27], it turns out that certain operators would only have a dimension-7 signature at low energy. Including dimension 7 operators for the $b \to s$ transition thus naturally enters an unprejudiced analysis of physics BSM.

In the next section, we shall derive a list of all the (on-shell) operators of mass-dimension 5, 6 and 7 which may intervene in the $b \to s$ EFT. We shall then detail the calculation of their ultraviolet (UV)-divergences at one-loop QCD order, before we proceed to the renormalization and establish the RGE's. The following section will be dedicated to the solution of these RGE's in some specific cases. Finally, we shall illustrate our discussion by presenting a concrete, if naive, case where our analysis of dimension 7 operators apply. A short conclusion will eventually summarize our achievements.

2 Operators of dimension $5 - 7$ in the $b \to s$ transition

2.1 List of operators

The first step of our analysis consists in establishing a list of all the operators of dimension 5, 6 and 7 intervening in the $b \to s$ transition (but Lorentz + gauge invariant!). For simplicity, all the fields shall be taken on-shell, i.e. satisfy their equation of motion: this will be sufficient, at least for the leading-order calculation that we aim at. For a discussion concerning the relevance of on-shell EFT's and in particular the question of applying only 'naive' classical equations of motions (dismissing ghosts and gauge-fixing terms), we refer the reader to [28]. We can obviously distinguish among three categories of operators: four-quark, two-quark + two lepton (semi-leptonic) and $\bar{b}s$-Gauge operators.
Let us start with four-fermion operators: the fermion fields already account for a mass-dimension 6, leaving room for at most one covariant (for gauge-invariance) derivative, when one restricts to operators of mass-dimension ≤ 7. Considering chiral fermions (that is, we include projectors $P_{L,R} \equiv \frac{1 \mp \gamma_5}{2}$ in the fermion products), there are at most three ways to contract the spinor algebra: $\mathbb{1}$ provides scalar currents, γ^μ, vector currents and $\sigma^{\mu\nu} \equiv [\gamma^\mu, \gamma^\nu]$,

tensor currents[4]; any higher combination of γ-matrices can be reduced down to those three cases (through the use of the Levi-Civita tensor $\varepsilon_{\mu\nu\rho\sigma}$ and identities of the Dirac algebra): note that γ_5 only gives a sign, when applied on chiral fermions. Moreover, we may always keep the b and s of the flavour transition within the same current: other combinations are made redundant by the Fierz identities (see e.g. Appendix A).
Those considerations allow us to construct the relevant three classes of dimension 6 operators:

1. scalar $(\bar{b}P_{L,R}s)(\bar{f}P_{L,R}f)$;
2. vector $(\bar{b}\gamma^{\mu}P_{L,R}s)(\bar{f}\gamma_{\mu}P_{L,R}f)$;
3. tensor $(\bar{b}\sigma^{\mu\nu}P_{L,R}s)(\bar{f}\sigma_{\mu\nu}P_{L,R}f)$. Note that only two chiral combinations are possible for the tensor currents: $L\times L$ and $R\times R$; the other combinations, $L\times R$ and $R\times L$, are identically zero.

One may then consider authentic dimension 7 operators by incorporating one covariant derivative in the fermion products. Two possibilities appear:

1. contracting the Lorentz index of D_μ with a vector current
2. contracting it with a tensor current, the second tensorial index being contracted with the second, vector current.

Note however that, using the equations of motion on fermions ($(\imath \not{D} - m_f)f = 0$) and realizing partial integrations (i.e. adding a total derivative to the lagrangian density), the resulting set of operators is largely redundant (together with the dimension 6 operators). Indeed, the second possibility which we mentioned ('$D^{\mu}\gamma^{\nu}\otimes\sigma_{\mu\nu}$') can always be reduced down to operators of the first class ('$D_\mu\otimes\gamma^{\mu}$') plus dimension 6 operators (multiplying fermion masses). Additionally, certain combinations of the operators of the first class reduce to dimension 6 terms. We thus retain only two kinds of linearly independant operators:

1. $[\bar{b}\imath(\overrightarrow{D}-\overleftarrow{D})^{\mu}P_{L,R}s](\bar{f}\gamma_{\mu}P_{L,R}f)$;
2. $(\bar{b}\gamma^{\mu}P_{L,R}s)[\bar{f}\imath(\overrightarrow{D}-\overleftarrow{D})_{\mu}P_{L,R}f]$.

For semi-leptonic operators, i.e. when f is a lepton ($f=l$), the analysis is essentially over.
Let us therefore focus on four-quark operators ($f=q$). One should then also consider the contraction of the colour indices. Two possibilities arise:

1. product of two colour-singlet currents: colour-indices contracted between the $\bar{b}$ and s on one side, $\bar{q}$ and q on the other;
2. product of two octet currents: colour-indices contracted between the $\bar{b}$ and q on one side, $\bar{q}$ and s on the other.

In the special cases where $q=b,s$, however, Fierz identities make this distinction superfluous, so that we may consider only singlet products then (refer to Appendix A). This is our final word for four-fermion operators.
We now turn to $\bar{b}s$-Gauge operators. The methodology follows that of [12] for the dimension 6 operators: one may simply write all the possibilities to include covariant derivatives (at most four for operators of dimension ≤ 7) within the fermionic current. Using partial integration and equations of motion, it turns out that all these covariant derivatives can be combined in field-strength tensors. One thus simply needs to consider the possibilities to combine the indices of these field-strength tensors with those of the fermionic current:

- When only one field-strength is present, it can only contract with a (Lorentz) tensor current, and, in the case of the QCD field strength $G^a_{\mu\nu}$, with a $SU(3)_c$ octet current.
- When two field-strengths are present, we may either contract their Lorentz indices together – thus reducing the fermionic current to a scalar – via the metric or a $\varepsilon_{\mu\nu\rho\sigma}$ tensor –, or contract two of their Lorentz indices with a tensor current (the other two through the metric, or equivalently $\varepsilon_{\mu\nu\rho\sigma}$). In the case where only one QCD field-strength is involved (among the two), one needs again a $SU(3)_c$ octet on the fermionic side. When two QCD field-strengths are involved ($G^a_{\mu\nu}G^b_{\rho\sigma}$), the color indices may be contracted together (δ^{ab}: colour-singlet), with a symmetric tensor ($\{T^a,T^b\}$) or with an antisymmetric tensor ($[T^a,T^b]$) (superposition of colour-octets + singlet), depending of the compatibility of these structures with the Lorentz form of the fermionic current.

[4]Note that the definition of $\sigma^{\mu\nu}$ which we adopt here for simplicity differs somewhat, by a factor $\imath/2$, from the one the reader may have encountered in the literature.

We now have all the ingredients to present the list of relevant operators. Note that their normalization is a priori free: our choice can be justified a posteriori to ensure that all the RGE's intervene at the same, leading-order in α_S. Note however that this choice may be slightly misleading, for instance in the SM, for reasons that we will discuss in the next section, when we solve the RGE's.

- $(\bar b s)(\bar l l)$ operators[5]:

$$\begin{cases} S^l_{L,R\;L,R} = \frac{\alpha}{\alpha_S} m_b(\bar b P_{L,R} s)(\bar l P_{L,R} l) \\ V^l_{L,R\;L,R} = \frac{\alpha}{\alpha_S} m_b(\bar b \gamma^\mu P_{L,R} s)(\bar l \gamma_\mu P_{L,R} l) \\ T^l_{L,R\;L,R} = \frac{\alpha}{\alpha_S} m_b(\bar b \sigma^{\mu\nu} P_{L,R} s)(\bar l \sigma_{\mu\nu} P_{L,R} l) \quad ; \quad \sigma^{\mu\nu} \equiv [\gamma^\mu, \gamma^\nu] \\ H^l_{L,R\;L,R} = \frac{\alpha}{\alpha_S} \left[\bar b \left(\imath \overrightarrow{\partial} - \imath \overleftarrow{\partial} + 2Q_d e A + 2 g_S T^a G^a\right)^\mu P_{L,R} s\right] (\bar l \gamma_\mu P_{L,R} l) \\ \tilde H^l_{L,R\;L,R} = \frac{\alpha}{\alpha_S} (\bar b \gamma^\mu P_{L,R} s) \left[\bar l \left(\imath \overrightarrow{\partial} - \imath \overleftarrow{\partial} + 2Q_l e A\right)_\mu P_{L,R} l\right] \end{cases} \tag{4}$$

- $(\bar b s)(\bar q q)$ operators[6]:

$$\begin{cases} S^q_{L,R\;L,R} = m_b(\bar b P_{L,R} s)(\bar q P_{L,R} q) \\ V^q_{L,R\;L,R} = m_b(\bar b \gamma^\mu P_{L,R} s)(\bar q \gamma_\mu P_{L,R} q) \\ T^q_{L,R\;L,R} = m_b(\bar b \sigma^{\mu\nu} P_{L,R} s)(\bar q \sigma_{\mu\nu} P_{L,R} q) \\ H^q_{L,R\;L,R} = \left[\bar b \imath \left(\overrightarrow{D} - \overleftarrow{D}\right)^\mu P_{L,R} s\right] (\bar q \gamma_\mu P_{L,R} q) \\ \tilde H^q_{L,R\;L,R} = (\bar b \gamma^\mu P_{L,R} s) \left[\bar q \imath \left(\overrightarrow{D} - \overleftarrow{D}\right)_\mu P_{L,R} q\right] \\ \mathcal{S}^q_{L,R\;L,R} = m_b(\bar b_\alpha P_{L,R} s_\beta)(\bar q_\beta P_{L,R} q_\alpha) \\ \mathcal{V}^q_{L,R\;L,R} = m_b(\bar b_\alpha \gamma^\mu P_{L,R} s_\beta)(\bar q_\beta \gamma_\mu P_{L,R} q_\alpha) \\ \mathcal{T}^q_{L,R\;L,R} = m_b(\bar b_\alpha \sigma^{\mu\nu} P_{L,R} s_\beta)(\bar q_\beta \sigma_{\mu\nu} P_{L,R} q_\alpha) \\ \mathcal{H}^q_{L,R\;L,R} = \left[\bar b_\alpha \imath \left(\overrightarrow{D} - \overleftarrow{D}\right)^\mu P_{L,R} s_\beta\right] (\bar q_\beta \gamma_\mu P_{L,R} q_\alpha) \\ \tilde{\mathcal{H}}^q_{L,R\;L,R} = (\bar b_\alpha \gamma^\mu P_{L,R} s_\beta) \left[\bar q_\beta \imath \left(\overrightarrow{D} - \overleftarrow{D}\right)_\mu P_{L,R} q_\alpha\right] \end{cases} \tag{5}$$

- $(\bar b s)$-Gauge operators:

$$\begin{cases} E_{L,R} = \frac{\imath e m_b^2}{4\pi\alpha_S} \bar b \sigma^{\mu\nu} P_{L,R} s\, F_{\mu\nu} \\ Q_{L,R} = \frac{\imath m_b^2}{g_S} \bar b \sigma^{\mu\nu} T^a P_{L,R} s\, G^a_{\mu\nu} \\ \mathcal{E}^S_{L,R} = \frac{\alpha}{\alpha_S} \bar b P_{L,R} s\, F_{\mu\nu} F^{\mu\nu} \\ \tilde{\mathcal{E}}^S_{L,R} = \imath \frac{\alpha}{\alpha_S} \bar b P_{L,R} s\, F_{\mu\nu} \tilde F^{\mu\nu} \\ \mathcal{E}^T_{L,R} = \frac{\alpha}{\alpha_S} \bar b \sigma^{\nu\rho} P_{L,R} s\, F_{\mu\nu} F^\mu_{\ \rho} \\ \mathcal{H}^S_{L,R} = \frac{e}{g_S} \bar b T^a P_{L,R} s\, F_{\mu\nu} G^{a\,\mu\nu} \\ \tilde{\mathcal{H}}^S_{L,R} = \frac{\imath e}{g_S} \bar b T^a P_{L,R} s\, F_{\mu\nu} \tilde G^{a\,\mu\nu} \\ \mathcal{H}^T_{L,R} = \frac{e}{g_S} \bar b \sigma^{\nu\rho} T^a P_{L,R} s\, F^\mu_{\ \nu} G^a_{\mu\rho} \\ \mathcal{Q}^D_{L,R} = \bar b P_{L,R} s\, G^a_{\mu\nu} G^{a\,\mu\nu} \\ \tilde{\mathcal{Q}}^D_{L,R} = \imath \bar b P_{L,R} s\, G^a_{\mu\nu} \tilde G^{a\,\mu\nu} \\ \mathcal{Q}^S_{L,R} = \bar b \frac{\{T^a, T^b\}}{2} P_{L,R} s\, G^a_{\mu\nu} G^{b\,\mu\nu} \\ \tilde{\mathcal{Q}}^S_{L,R} = \imath \bar b \frac{\{T^a, T^b\}}{2} P_{L,R} s\, G^a_{\mu\nu} \tilde G^{b\,\mu\nu} \\ \mathcal{Q}^T_{L,R} = \bar b \sigma^{\nu\rho} \frac{[T^a, T^b]}{2} P_{L,R} s\, G^a_{\mu\nu} G^{b\,\mu}_{\ \ \ \rho} \end{cases} \tag{6}$$

This list determines the effective hamiltonian of our EFT (the generic notation O^i spans the whole list):

$$\mathcal{H}_{eff}^{(\dim \leq 7)} = \sum_i C_i(\mu) O^i(\mu) + h.c. \tag{7}$$

[5]We repeat that, for the tensor operators, only the $L \times L$ and $R \times R$ combinations are relevant, which may not be obvious from our notation.

[6]We discard $\mathcal{S}, \mathcal{V}, \mathcal{T}, \mathcal{H}, \tilde{\mathcal{H}}^q$ for $q = b, s$ since they reduce to $S, V, T, H, \tilde H^q$ due to Fierz transformations which are provided in Appendix A. The colour indices α, β of the fermions are displayed when not trivially contracted. Note finally that the ambiguous notation '$\overrightarrow{D}_\mu q_\alpha$' stands actually for $(D_\mu q)_\alpha \ldots$

Summing over all the flavours and chiralities, we count 251 terms.
For the purpose of illustration, let us sketch the typical matching conditions that one could expect for these operators in a Minimal-Flavour-Violating model (the CKM matrix elements are written as $V_{qq'}$). Only the operators $(\bar{b}s)(\bar{q}q)$ with $q = c, u$ would receive a contribution at tree-level: $C_{q=c,u} \sim \frac{G_F V_{qs} V_{qb}^*}{m_b}$ (the case $q = u$ may be neglected because of the CKM suppression). All the other contributions would arise at the loop-level, $C_{\text{dim } 6} \sim \frac{\alpha_S}{4\pi} \frac{G_F V_{ts} V_{tb}^*}{m_b}$ and $C_{\text{dim } 7} \sim \frac{\alpha_S}{4\pi} \frac{G_F V_{ts} V_{tb}^*}{m_b} \frac{m_b}{M_W}$ for the operators present at dimension 6 and 7 respectively.

2.2 UV-divergent amplitudes involving the dimension $5-7$ operators

Before renormalizing the EFT, we compute the divergences associated with QCD loops. We perform this calculation in the most naive conceivable way: in the Feynman-t'Hooft gauge and dimensional regularization $D = 4 - 2\varepsilon$; we keep only the divergent terms $\left[2 - \frac{D}{2}\right]^{-1}$. Note that we will assume that the couplings C_i come together with an electric charge factor e whenever a photon or a lepton appear in the external lines: in this fashion $(\bar{b}s)(\bar{l}l)$ shall be regarded as electroweakly suppressed with respect to $(\bar{b}s)(\bar{q}q)$, rejecting $(\bar{b}s)(\bar{l}l)$ UV-contributions to $(\bar{b}s)(\bar{q}q)$ operators to a subdominant order. The computation can be organized in 'blocks':

- $(\bar{b}s)(\bar{l}l)$ contributions to the $(\bar{b}s)(\bar{l}l)$ amplitude: see Fig.1

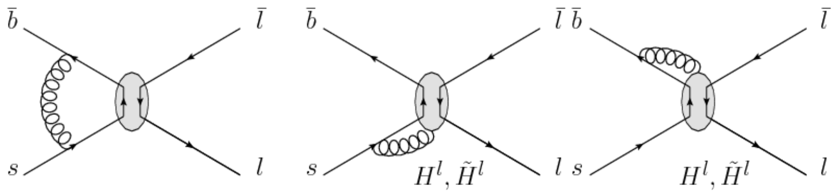

Figure 1: Contributions from the $(\bar{b}s)(\bar{l}l)$ operators to the $\bar{b}s \to \bar{l}l$ amplitude. Here as in the following plots, the grey blob represents a vertex associated with the operators of mass-dimension $5-7$. The subscript '$H^l, \tilde{H}^l$' under some of the diagrams indicates that such diagrams are relevant only for this kind of operators. Feynman diagrams are plotted with JaxoDraw [29].

- $(\bar{b}s)(\bar{q}q)$ contributions to the $(\bar{b}s)$-Gauge amplitude.

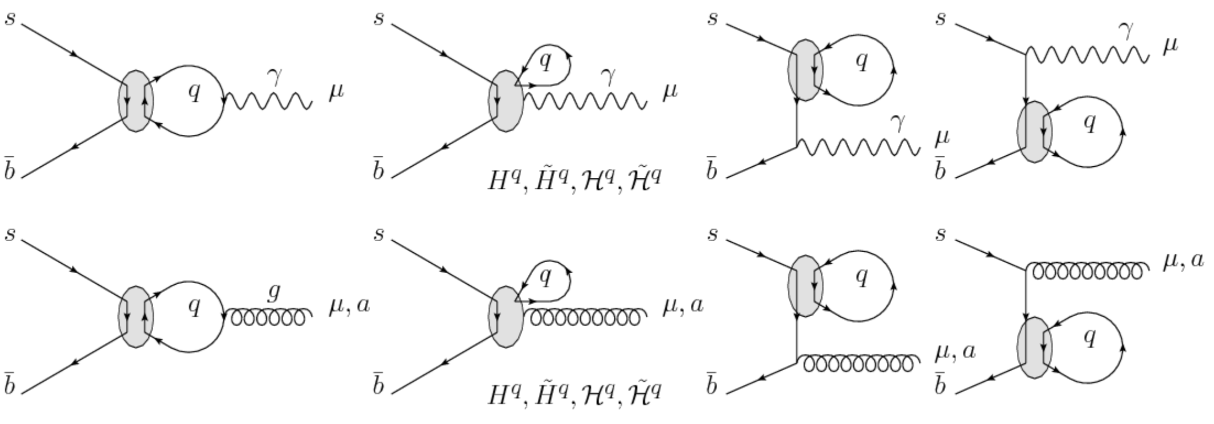

Figure 2: Contributions from the $(\bar{b}s)(\bar{q}q)$ operators to the $\bar{b}s \to \gamma/g$ amplitude. Again, as in the following figures, the label '$H^q, \tilde{H}^q, \mathcal{H}^q, \tilde{\mathcal{H}}^q$' indicates that the corresponding diagrams intervene for such operators only.

  Such contributions involve a quark loop. While the external fermions are explicitly taken on-shell, through the application of their equations of motion, explicit off-shell terms appear for the external photons and gluons: such terms must organize as an equation of motion for the gauge-boson line, which provides us with a cross-check of our calculation. They do not matter for the renormalization so that we will not keep them explicitly in the divergent amplitudes. The amplitudes $(\bar{b}s) - \gamma^*$ or $(\bar{b}s) - g^*$ are relevant for the divergent amplitudes $(\bar{b}s)(\bar{l}l)$, $(\bar{b}s)(\bar{q}'q')$ and $(\bar{b}s)(gg)$ though, which we will present later.

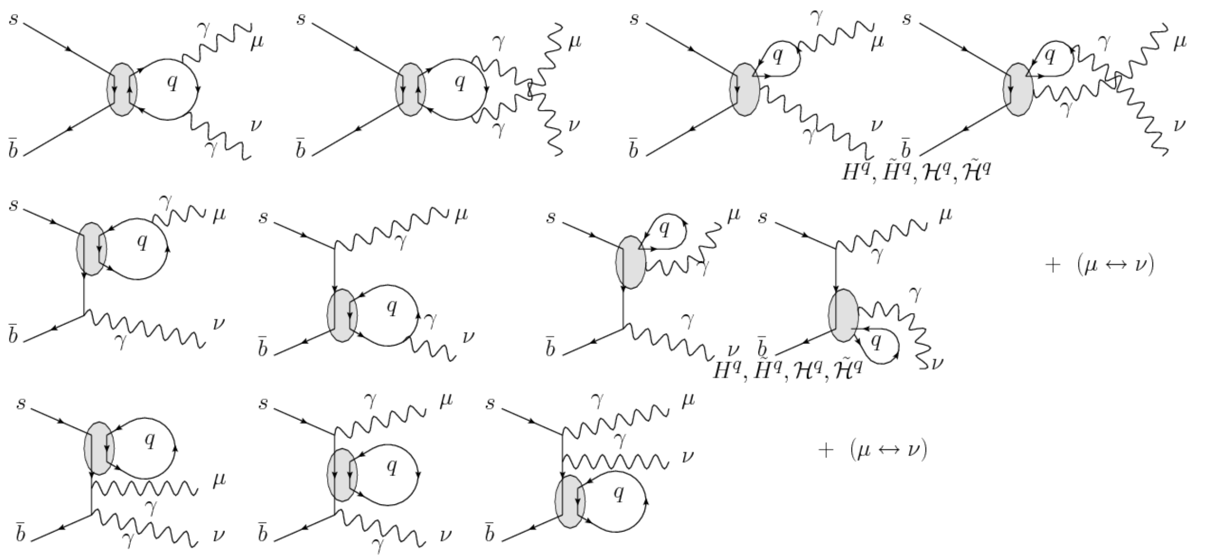

Figure 3: Contributions from the $(\bar{b}s)(\bar{q}q)$ operators to the $\bar{b}s \to 2\gamma$ amplitude. $+(\mu \leftrightarrow \nu)$ signals that one must add the 'mirror' diagrams exchanging the two photon lines.

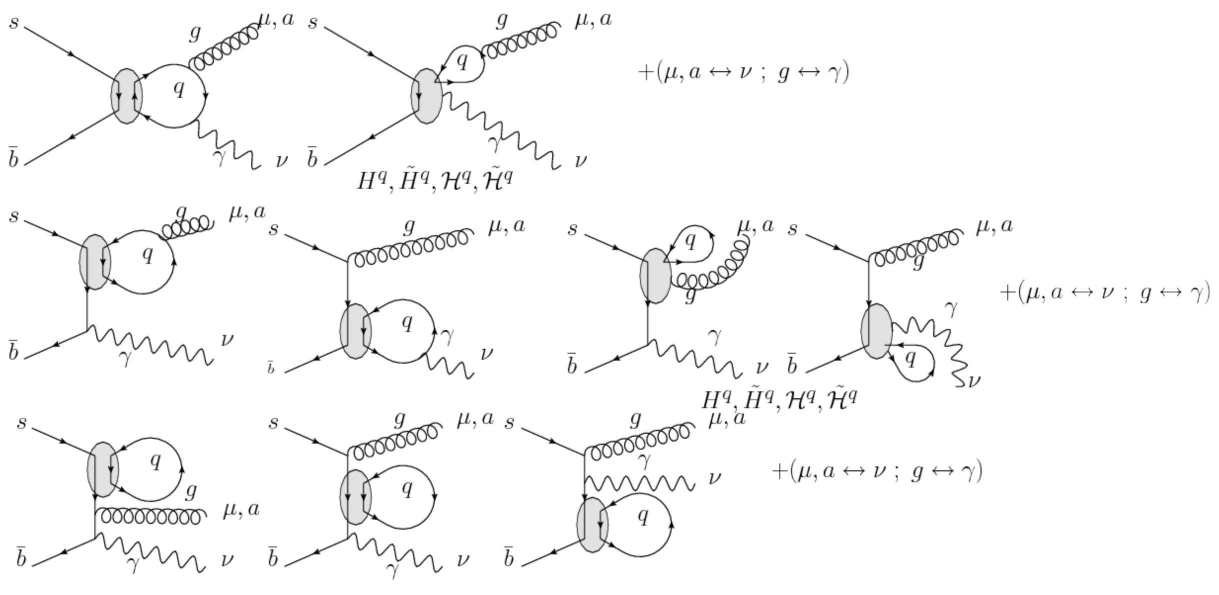

Figure 4: Contributions from the $(\bar{b}s)(\bar{q}q)$ operators to the $\bar{b}s \to \gamma g$ amplitude.

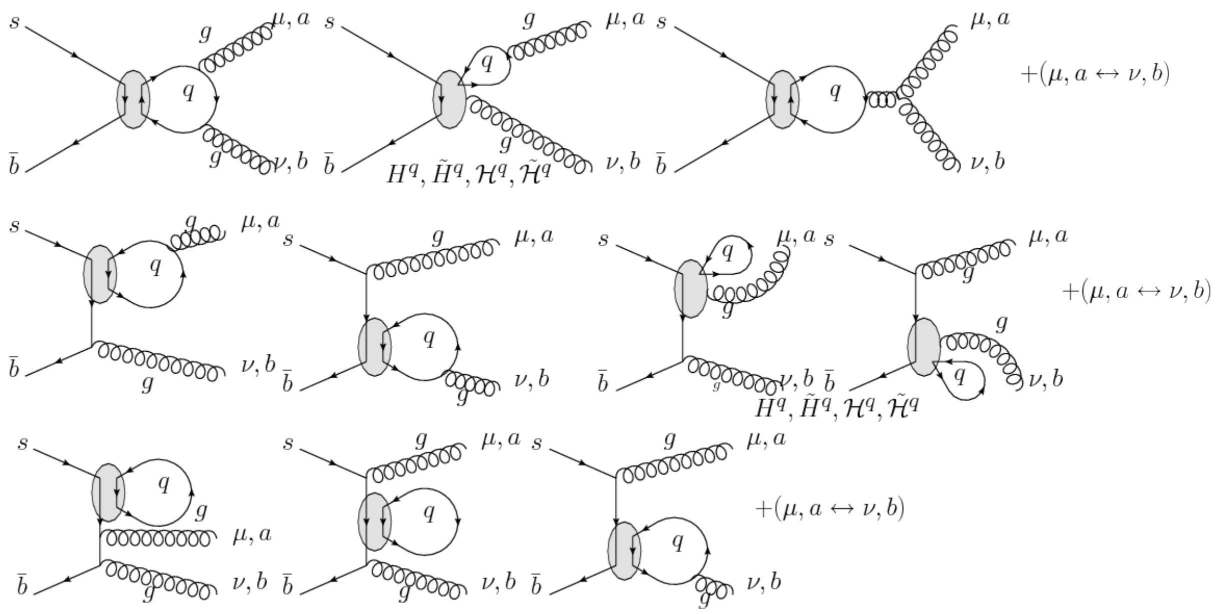

Figure 5: Contributions from the $(\bar{b}s)(\bar{q}q)$ operators to the $\bar{b}s \to 2g$ amplitude.

The contributions to the amplitudes $\bar{b}s \to \gamma/g$ are shown in Fig.2. The diagrams intervening in $\bar{b}s \to 2\gamma/\gamma g/2g$ are depicted in Fig.3,4,5. Note that in the case of the dimension 6 four-quark operators, no contribution to the $\bar{b}s - VV'$ (where $V, V' \in \{\gamma, g\}$) operators is expected: the dimension 6 basis is stable and does not require dimension 7 counterterms. The corresponding amplitudes in Fig.3,4,5 therefore provide a simple check of the matching conditions determined in Fig.2. For the authentic dimension 7 operators, however, the calculation of the $\bar{b}s - VV'$ amplitudes is fully relevant.

The operators $(\bar{b}s)(\bar{q}q)$ with $q = b, s$ deserve a particular attention. Beyond the 's-channel' diagrams, similar to those of the other $(\bar{b}s)(\bar{q}q)$ operators, they indeed allow for a 't-channel' contribution: these can be viewed as '$\frac{\pi}{2}$-rotated' contributions associated with the $\mathcal{S}^q$, $\mathcal{V}^q$, $\mathcal{T}^q$, $\mathcal{H}^q$, $\tilde{\mathcal{H}}^q$ operators, which, in the case of $q = b, s$, coincide with S^q, V^q, T^q, H^q, $\tilde{H}^q$, via Fierz identities. Instead of computing the new 't-channel' contributions[7], we thus use our generic result for the $\mathcal{S}^q$, $\mathcal{V}^q$, $\mathcal{T}^q$, $\mathcal{H}^q$, $\tilde{\mathcal{H}}^q$ operators and project it on S^q, V^q, T^q, H^q, $\tilde{H}^q$ with the Fierz matrices $\tilde{V}^{b,s}_{\text{Fierz}}$ exchanging the two bases and defined in Appendix A. A factor -1 appears in this process corresponding to the anticommutation of two fermion fields / the transformation of an open fermion loop into a closed one.

- $(\bar{b}s)(\bar{q}q)$ contributions to the $(\bar{b}s)(\bar{f}f)$ amplitude.

The contributions to the $\bar{b}s \to \bar{l}l$ amplitude proceed only from the exchange of an off-shell photon, as depicted on the first diagram of Fig.6. For the $\bar{b}s \to \bar{q}'q'$ amplitude, contributions originate similarly from the exchange of an off-shell gluon (note that the photon exchange would be of a higher-order). Additional 'diagonal' contributions are obtained for $q = q'$ through the dressing of the $(\bar{b}s)(\bar{q}q)$ vertex by a gluon line. All the corresponding diagrams are depicted in Fig.6.

[7] We, in fact, checked this relation explicitly.

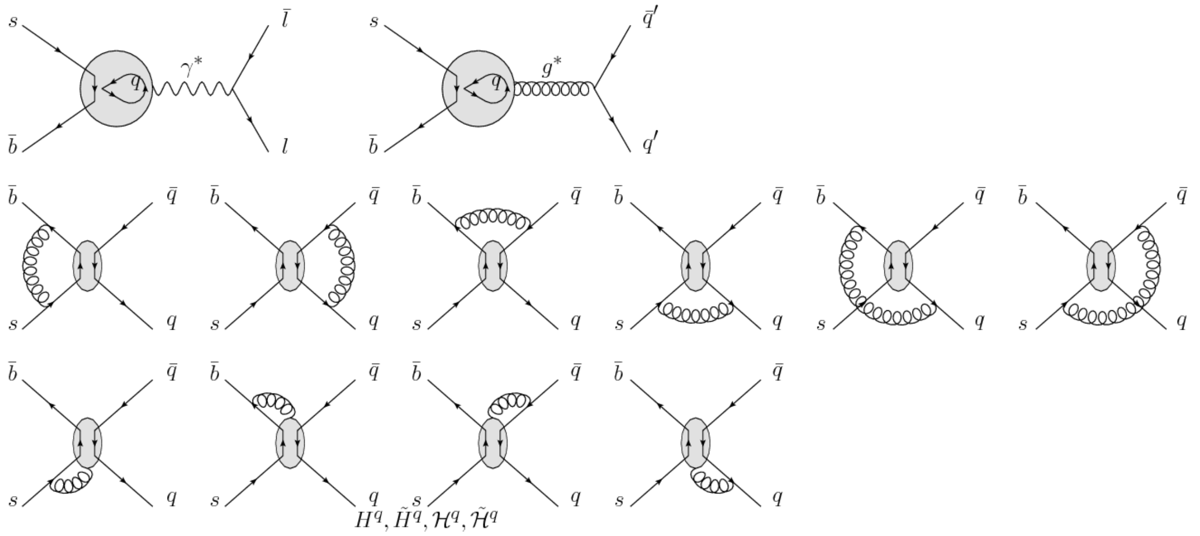

Figure 6: Contributions from the $(\bar{b}s)(\bar{q}q)$ operators to the $\bar{b}s \to \bar{l}l$ and $\bar{b}s \to \bar{q}q$ amplitudes. The grey blob with a quark loop stands for all possible $(\bar{b}s)(\bar{q}q)$ contributions to an off-shell photon/gluon: refer to Fig.2.

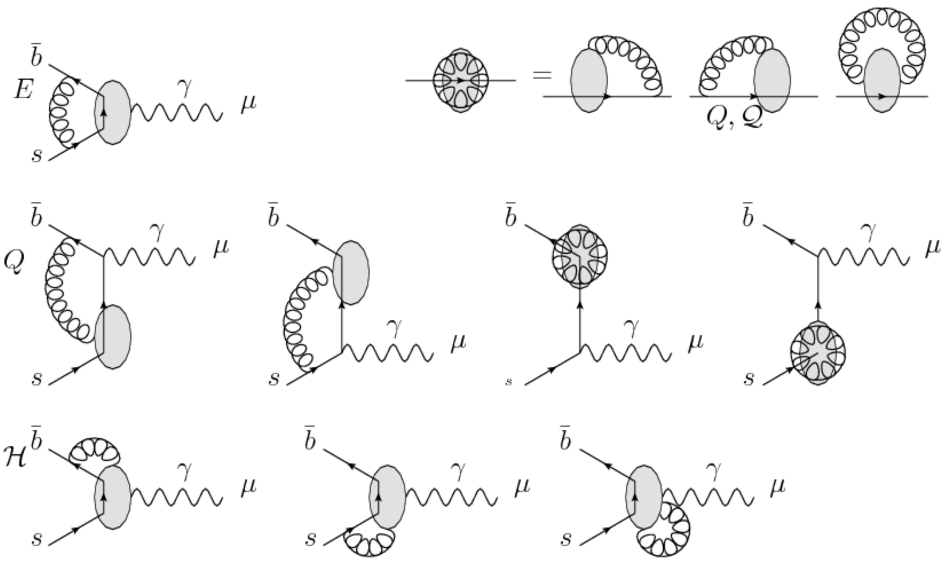

Figure 7: Contributions from the $(\bar{b}s)$-Gauge operators (E, Q and $\mathcal{H} \leftrightarrow \mathcal{H}^S, \tilde{\mathcal{H}}^S, \mathcal{H}^T$) to the $\bar{b}s \to \gamma$ amplitude. We also define the $b \to s$ self-energy associated with the operators Q (and $\mathcal{Q} \leftrightarrow \mathcal{Q}^D$, $\tilde{\mathcal{Q}}^D$, $\mathcal{Q}^S$, $\tilde{\mathcal{Q}}^S$, $\mathcal{Q}^T$) and depicted on this figure, as well as the following ones, by a fermion line with a grey blob and a gluon bubble.

- $(\bar{b}s)$-Gauge contributions to the $(\bar{b}s)$-Gauge amplitudes.

  Once again, off-shell photons and gluons appear within the calculation of these amplitudes, offering a nice crosscheck of the result, given that they must combine into an equation of motion for the corresponding field. Although such terms will not matter for the renormalization, $\bar{b}s \to \gamma^*$ and $\bar{b}s \to g^*$ will be relevant for $\bar{b}s \to \bar{l}l/\bar{q}q/gg$. Another check, particularly in the case where two gauge bosons appear in the final state, is simply the projectibility of the result on the basis of operators that we have defined: indeed the results typically reach this form only after the summation of several diagrams, the coefficients and colour-factors of which must combine in the appropriate way. Note that only 'authentic' dimention 7 operators need to be included in the amplitudes $\bar{b}s \to \gamma\gamma/\gamma g/gg$: the presence of E or Q in the divergences of such amplitudes (substracting however the counterterms of E and Q) would signal an instability of the dimension 6 basis, which would require dimension 7 counterterms. This evidently does not occur.

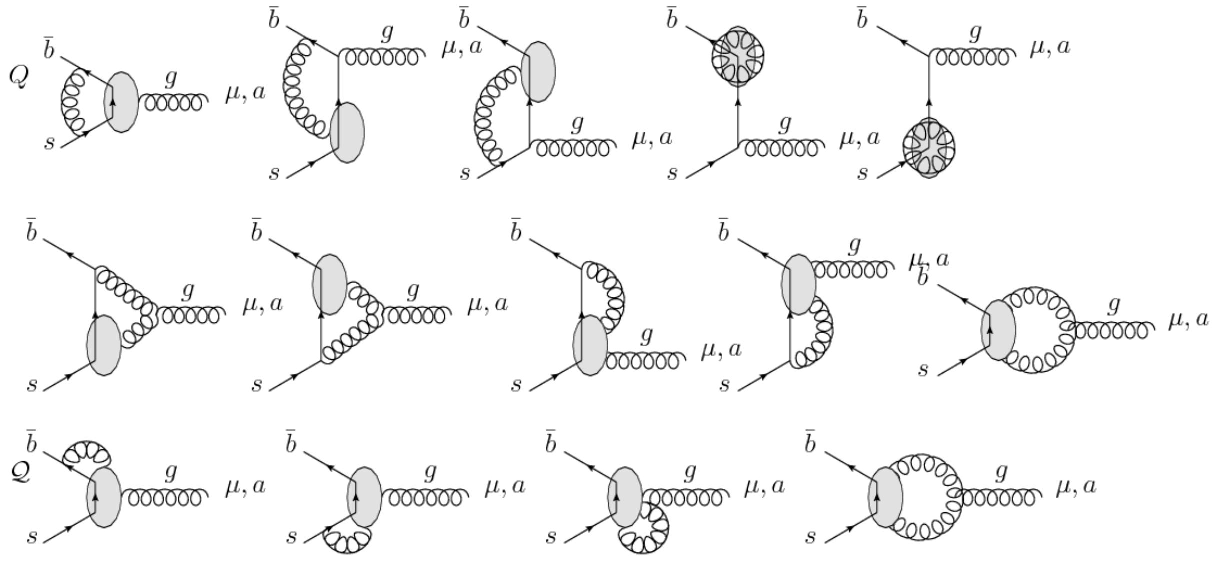

Figure 8: Contributions from the $(\bar{b}s)$-Gauge operators (Q and $\mathcal{Q}$) to the $\bar{b}s \to g$ amplitude.

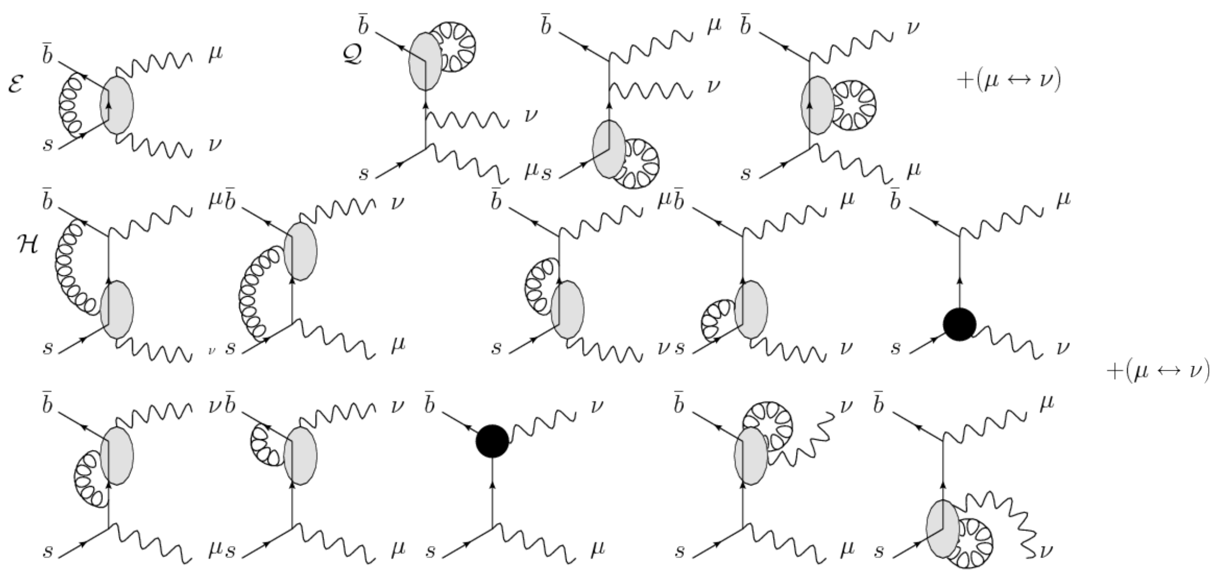

Figure 9: Contributions from the $(\bar{b}s)$-Gauge operators ($\mathcal{E} \leftrightarrow \mathcal{E}^S, \tilde{\mathcal{E}}^S, \mathcal{E}^T$ and $\mathcal{H}$) to the $\bar{b}s \to 2\gamma$ amplitude. The black blobs signal a counterterm from the $\bar{b}s\gamma$ or $\bar{b}sg$ amplitudes.

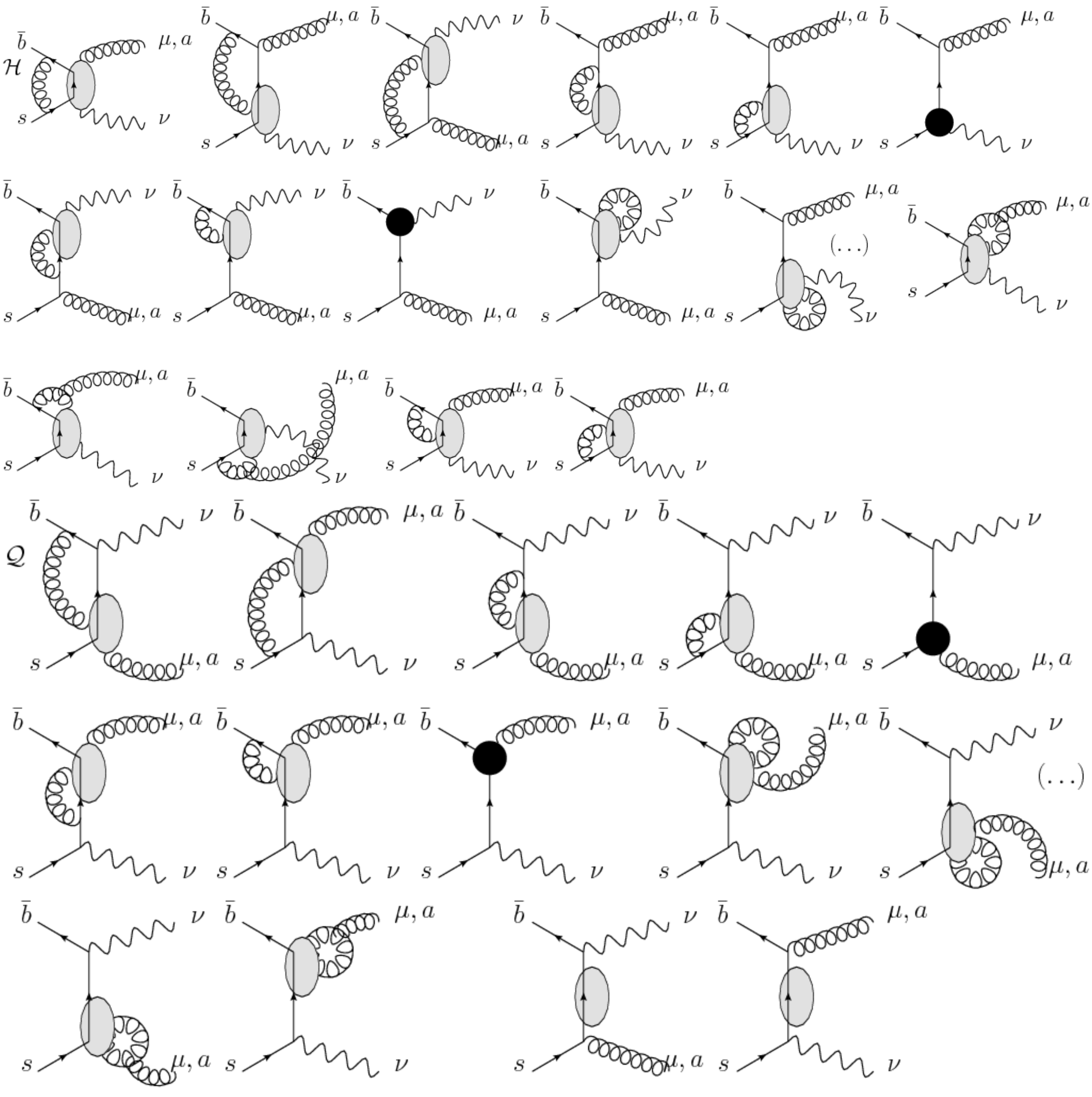

Figure 10: Contributions from the $(\bar{b}s)$-Gauge operators ($\mathcal{H}$ and $\mathcal{Q}$) to the $\bar{b}s \to \gamma g$ amplitude. The $(\ldots)$ stand for additional diagrams involving a gluon tadpole, which give vanishing contributions.

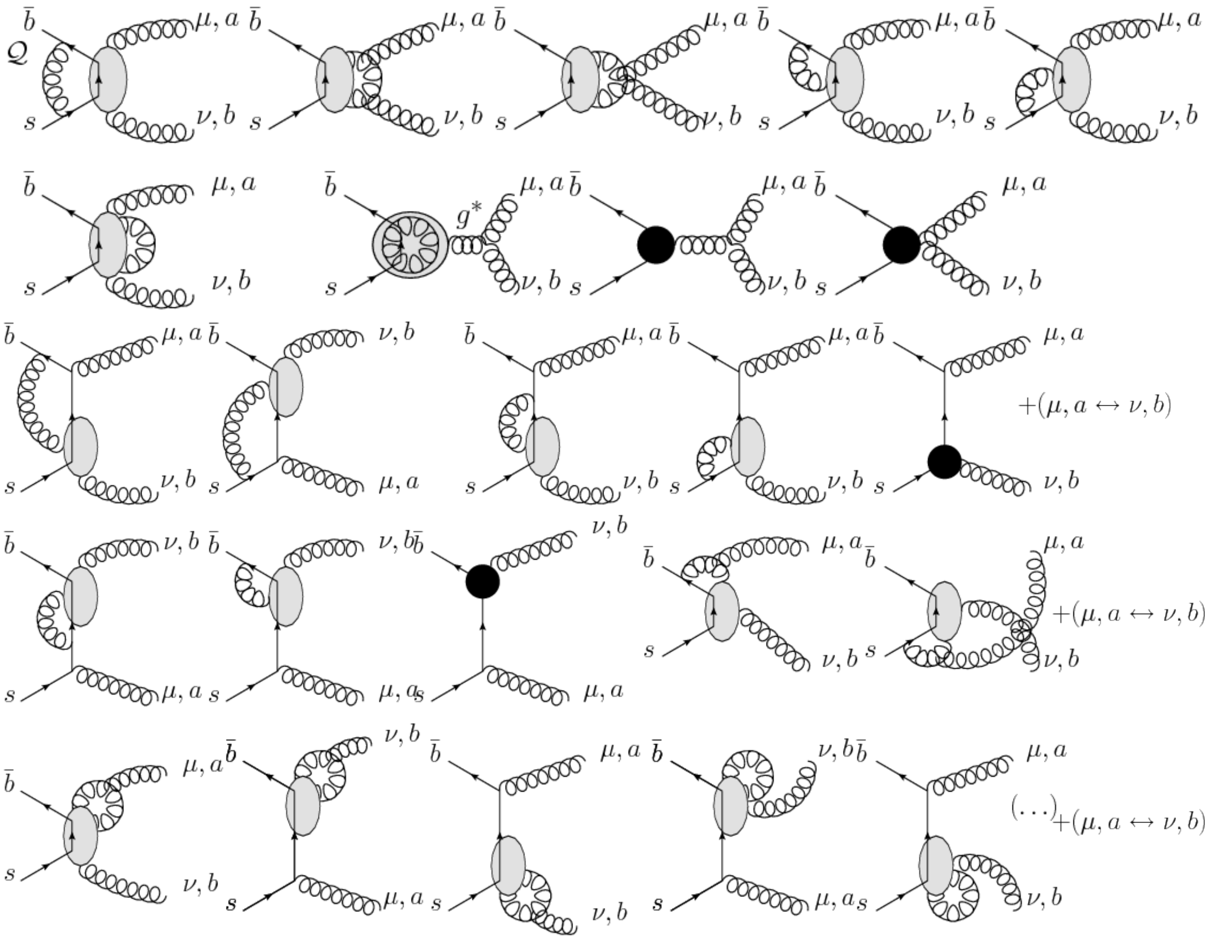

Figure 11: Contributions from the $(\bar{b}s)$-Gauge operators $(\mathcal{Q})$ to the $\bar{b}s \to 2g$ amplitude. The grey blob with an inscribed gluon bubble and attached to a photon/gluon line stands for all the contributions to off-shell photon/gluon production: refer to Fig.7,8.

- $(\bar{b}s)$-Gauge contributions to the $(\bar{b}s)(\bar{f}f)$ amplitudes. Such contributions proceed from off-shell photon or gluon exchanges or a gluon 'pseudo-box'[8] and are depicted on Fig.12.

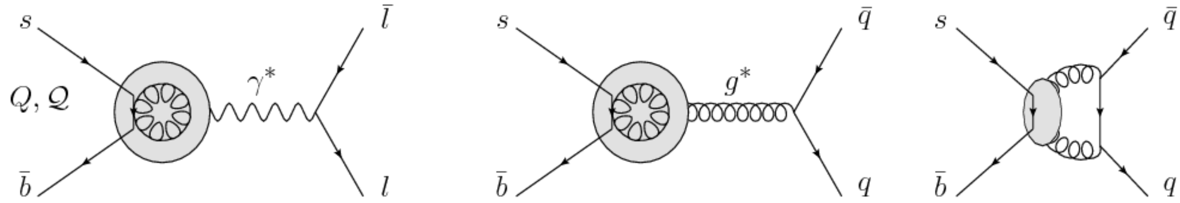

Figure 12: Contributions from the $(\bar{b}s)$-Gauge operators (Q, $\mathcal{H}$ and $\mathcal{Q}$) to the $\bar{b}s \to \bar{l}l/\bar{q}q$ amplitude.

Finally, we may cast all these divergent (on-shell) amplitudes into the following matrix form:

$$\text{div} = \frac{\imath}{4\pi\left(2-\frac{D}{2}\right)}(C)\mathcal{M}(\Gamma) \tag{8}$$

where $(C) = (C_{Sl}^{L,R\,L,R}, \ldots)$ is a row vector[9] collecting all the couplings of the effective Hamiltonian and (Γ) is a column vector constituted by all the form-factors associated to the operators and intervening in the amplitudes that we have just presented (we only need one form-factor per operator since we have not considered redundant amplitudes such as $\bar{b}s \to \bar{f}f\gamma$, etc.). $\mathcal{M}$ depends only on fermion masses, gauge couplings and charges.

2.3 Renormalization of the operators

2.3.1 Basics of the renormalization of the EFT in the MS-scheme

Let us recall a few basic ingredients[10] of the renormalization procedure for our EFT in the MS-scheme.
Dismissing the effective hamiltonian for the time being, the (dim≤ 4) EFT is a renormalizable QED$\times$QCD model. We neglect the QED loop-effects here, since $\alpha \ll \alpha_S$, hence focus on a QCD renormalization only. The fermion wave-function and mass renormalization constants, Z_f and Z_{m_f} respectively, are determined by requiring that the counterterm contribution to the f self-energy (with p external momentum), $\Sigma_f^{CT}(p) = \imath(Z_f - 1)\not{p} - \imath(Z_f Z_{m_f} - 1)$, cancels the divergent loop contributions: $\Sigma_f^{\text{loop div}}(p) = 0$ (leptons) or $-\frac{\imath C_2(3)\alpha_S}{4\pi\left(2-\frac{D}{2}\right)}[-\not{p} + 4m_f]$ (quarks), where $C_2(3) = \frac{4}{3}$ denotes the Casimir operator of $SU(3)_c$ in the fundamental representation:

$$Z_l = 1 = Z_{m_l} \qquad ; \qquad \begin{cases} Z_q = 1 - \frac{C_2(3)\alpha_S}{4\pi\left(2-\frac{D}{2}\right)} + O(\alpha_S^2) \\ Z_{m_q} = 1 - \frac{3C_2(3)\alpha_S}{4\pi\left(2-\frac{D}{2}\right)} + O(\alpha_S^2) \end{cases} \tag{9}$$

The gluon self energy (involving gluons, quarks and ghosts) similarly determines the gluon renormalization constant: in the Feynman gauge, $Z_3 \simeq 1 - \frac{\alpha_S}{4\pi\left(2-\frac{D}{2}\right)}\left[\frac{2}{3}n_F - \frac{5}{3}N_c\right]$, with $n_F = 5$ the number of quark flavours and $N_c = 3$ the number of colours. From the quark-gluon vertex, one renormalizes the strong coupling constant g_S, resulting in the RGE (from the scale-independence of the bare coupling) for $\alpha_S \equiv \frac{g_S^2}{4\pi}$, with μ the renormalization scale:

$$\frac{d\alpha_S}{d\ln\mu} = -2\left(2-\frac{D}{2}\right)\alpha_S + \beta(\alpha_S) \;\; ; \;\; \beta(\alpha_S) = -\frac{2\beta_0}{4\pi}\alpha_S^2 + O(\alpha_S^3) \;\; ; \;\; \beta_0 = \frac{11N_c - 2n_F}{3} = \frac{23}{3}$$
$$\Rightarrow \quad \alpha_S(\mu) \simeq \frac{\alpha_S(\mu_0)}{1 + \alpha_S(\mu_0)\frac{2\beta_0}{4\pi}\ln\frac{\mu}{\mu_0}} \tag{10}$$

[8]By which we mean the last diagram of Fig.12.
[9]We refrain from writing this row vector with a transposition symbol (i.e. $(C)^T$), not to make already-heavy notations as will appear in the following sections completely unreadable.
[10]Those are well-known basics of the QCD renormalization and are available in countless textbooks. We mention them only for the sake of completeness as well as clarity in our notations.

The bare fields $Z_f f$ as well as the bare masses $Z_{m_f} m_f$ must be scale independent (note that the μ-dependence enters Z_f and Z_{m_f} indirectly through α_S). One deduces the quark-mass running:

$$\frac{dm_q}{d\ln\mu} = -6C_2(3)\frac{\alpha_S}{4\pi}m_q + O(\alpha_S^2) \;\Leftrightarrow\; \frac{dm_q}{d\ln\alpha_S} = \frac{3C_2(3)}{\beta_0}m_q + O(\alpha_S) \;\Rightarrow\; \frac{m_q(\mu)}{m_q(\mu_0)} = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{3C_2(3)}{\beta_0}} \tag{11}$$

Let us come back to the effective hamiltonian. The loop-divergences in the amplitudes, Eq.(8), must be compensated by the operator counterterm contributions: $\imath(C)[Z^C-\mathbb{1}]Z_\Gamma(\Gamma)$, with Z^C the operator-renormalization matrix and (the diagonal) Z_Γ the renormalization constant of the fields (and normalization) entering the form factor Γ. This determines: $Z^C = [\mathbb{1} - \frac{\mathcal{M}}{4\pi(2-\frac{D}{2})}]Z_\Gamma^{-1}$. Scale-independence of the bare couplings $(C)Z^C$ (by definition of Z_Γ, $Z_\Gamma O_\Gamma$ is scale-independent) leads to the RGE for the couplings (C):

$$\frac{d(C)}{d\ln\mu} = (C)\gamma_C \quad ; \quad \gamma_C \equiv -\frac{d[Z^C]}{d\ln\mu}(Z^C)^{-1} \quad ; \quad Z^C = [\mathbb{1} - \frac{\mathcal{M}}{4\pi\left(2-\frac{D}{2}\right)}]Z_\Gamma^{-1} \tag{12}$$

where the μ-dependence of Z^C originates from the dependence of $\mathcal{M}$ on g_S and quark-masses, as well as from the renormalization constants in Z_Γ. We have introduced the anomalous dimension matrix γ_C.
Considering the normalization of the operators that we introduced at the begining of this section, one may factor out harmoniously $C_2(3)\alpha_S$ in the matrix $\mathcal{M}$, leading to:

$$\text{div} = \frac{\imath C_2(3)\alpha_S}{4\pi\left(2-\frac{D}{2}\right)}(C)\tilde{\mathcal{M}}(\Gamma) \tag{13}$$

where $\tilde{\mathcal{M}}$ is a constant matrix (depending only on quark-mass ratios $\frac{m_q}{m_b}$).
At this leading order in QCD, using Eq.10, Eq.12 may be written explicitly as:

$$\begin{aligned}\frac{d(C)}{d\ln\mu} &= \frac{2\alpha_S C_2(3)}{4\pi}(C)\left[\tilde{\mathcal{D}}-\tilde{\mathcal{M}}\right] + O(\alpha_S^2) \;\Leftrightarrow\; \frac{d(C)}{d\ln\alpha_S} = -\frac{C_2(3)}{\beta_0}(C)\left[\tilde{\mathcal{D}}-\tilde{\mathcal{M}}\right] + O(\alpha_S)\\ &\Rightarrow \qquad \gamma_C = \frac{2\alpha_S C_2(3)}{4\pi}\left[\tilde{\mathcal{D}}-\tilde{\mathcal{M}}\right] + O(\alpha_S^2)\end{aligned} \tag{14}$$

where we have introduced the diagonal matrix $\frac{2\alpha_S C_2(3)}{4\pi}\tilde{\mathcal{D}} \equiv \frac{dZ_\Gamma}{d\ln\mu}$. The corresponding scaling may be extracted from the field renormalization factors ($Z_q^{1/2}$ for each external quark and $Z_3^{1/2}$ for the external gluons) and the normalization factors (inserting g_S, α_S, m_b).

2.3.2 Renormalization Group Equations for the operator basis

We can now explicitly extract the RGE's for the operators under consideration. The operators are ordered as above in Eq.(4,5,6). Moreover, as far as the chiralities are concerned, we use the ordering LL, LR, RR, RL for four-fermion operators (LL, RR for the tensors) and L, R for the $\bar{b}s$-Gauge operators: we will not write them down explicitly in the following, so as to keep notations as tractable as possible, but remember that the coefficients come together with one or two chirality indices. The anomalous dimension matrix can be split in several blocks, corresponding to the various types (four-quark, semi-leptonic, $\bar{b}s$-gauge) of operators:

$$\gamma_C = \frac{2\alpha_S C_2(3)}{4\pi}\begin{bmatrix}\tilde{\mathcal{D}}^{ll}-\tilde{\mathcal{M}}^{ll} & 0 & 0\\ -\tilde{\mathcal{M}}^{ql} & \tilde{\mathcal{D}}^{qq}-\tilde{\mathcal{M}}^{qq} & -\tilde{\mathcal{M}}^{qg}\\ -\tilde{\mathcal{M}}^{gl} & -\tilde{\mathcal{M}}^{gq} & \tilde{\mathcal{D}}^{gg}-\tilde{\mathcal{M}}^{gg}\end{bmatrix} \quad ; \quad (C) = (C^l, C^q, C^g) \tag{15}$$

where the meaning of the subindices l, q and g should be transparent. For the sake of clarity, we choose to present the various blocks along with the specific RGE that they affect, separating the diagonal scaling contributions from the UV-divergent diagrams of section 2.2. However, should one wish to implement the whole 251×251 γ_C matrix directly without considering our detailed study, the corresponding results are gathered in Appendix B.

- $(\bar{b}s)(\bar{l}l)$ couplings[11]:

[11] Note that operators with different lepton flavours do not mix, justifying the use of only one index l.

$$\frac{d(C^l)}{d\ln\alpha_S} = -\frac{C_2(3)}{\beta_0}\left\{(C^l)\left[-\frac{1}{4}\mathrm{diag}(7\mathbb{1},7\mathbb{1},7\mathbb{1},19\mathbb{1},19\mathbb{1}) - \tilde{\mathcal{M}}^{ll}\right] - (C^g)\tilde{\mathcal{M}}^{gl} - (C^q)\tilde{\mathcal{M}}^{ql}\right\} + O(\alpha_S) \tag{16}$$

The matrices $\tilde{\mathcal{M}}^{ll,gl,ql}$ must be extracted from our calculation of the divergent contributions[12]:

$$\tilde{\mathcal{M}}^{ll} = \begin{bmatrix} 4\mathbb{1} & 0 & 0 & 0 & 0 \\ 0 & \mathbb{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2\left(\mathbb{1}+\frac{m_s}{m_b}\Sigma\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbb{1} \end{bmatrix} \quad ; \quad \Sigma \equiv \begin{matrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{matrix} \tag{17}$$

$$\tilde{\mathcal{M}}^{gl} = -\frac{1}{3}Q_l \begin{array}{c} \left[\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ & & \cdots & & \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \Xi + \frac{m_s}{m_b}\Phi & 0 & 0 & 0 \\ 0 & 2[\Xi' + \frac{m_s}{m_b}\Phi'] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ & & \cdots & & \\ 0 & 0 & 0 & 0 & 0 \end{matrix}\right] \end{array} \begin{matrix} \leftarrow E \\ \cdots \\ \leftarrow \mathcal{E}^T \\ \leftarrow \mathcal{H}^S \\ \leftarrow \tilde{\mathcal{H}}^S \\ \leftarrow \mathcal{H}^T \\ \cdots \\ \leftarrow \mathcal{Q}^T \end{matrix} \quad ; \quad \begin{cases} \Xi \equiv \begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{matrix} \\ \Phi \equiv \begin{matrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{matrix} \\ \Xi' \equiv \begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{matrix} \\ \Phi' \equiv \begin{matrix} 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 \end{matrix} \end{cases} \tag{18}$$

$$\tilde{\mathcal{M}}^{ql} = \frac{3}{2}Q_l Q_q[\mathbb{1}; -\tilde{V}^q_{\text{Fierz}}]_{(q=b,s)} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{m_b}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{3m_b}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \end{bmatrix} \quad ; \quad \tilde{\Sigma} \equiv \begin{matrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{matrix} \tag{19}$$

We define the object $[\mathbb{1}; -\tilde{V}^q_{\text{Fierz}}]_{(q=b,s)}$ as follows: when $q \neq b, s$ it is simply the (36×36) identity in the space defined by the corresponding $(\bar{b}s)(\bar{q}q)$ operators; when $q = b, s$, then it is a (18×36) matrix built with the (18×18) identity in the left-hand subblock and the (18×18) transition-matrix $-\tilde{V}^q_{\text{Fierz}}$, defined in Appendix A, in the right-hand subblock. Its origin is related to the facts that, for $q = b, s$, Fierz identities shorten the list of independent operators (compared to the cases $q \neq b, s$), and that the additional diagrams 'in the t-channel', which open for $q = b, s$, can be related to contributions that the redundant (omited) operators would have 'in the s-channel': refer to our discussion in section 2.2.

- $(\bar{b}s)(\bar{q}q)$ couplings[13]:

$$\begin{aligned} \frac{d(C^q)}{d\ln\alpha_S} = -\frac{C_2(3)}{\beta_0}\Bigg\{(C^{q'})\Bigg[&\delta^{q'q}[\mathbb{1},0]_{(q'=b,s)}\mathrm{diag}(5\mathbb{1},5\mathbb{1},5\mathbb{1},2\mathbb{1},2\mathbb{1},5\mathbb{1},5\mathbb{1},5\mathbb{1},2\mathbb{1},2\mathbb{1})\begin{bmatrix}\mathbb{1}\\0\end{bmatrix}_{(q=b,s)} - \tilde{\mathcal{M}}^{q'q}\Bigg] \\ &- (C^g)\tilde{\mathcal{M}}^{gq}\Bigg\} + O(\alpha_S) \end{aligned} \tag{20}$$

The objects $[\mathbb{1},0]_{(q=b,s)}$ and $\begin{bmatrix}\mathbb{1}\\0\end{bmatrix}_{(q=b,s)}$ are again defined as the (36×36) identity when $q \neq b, s$ and the $(18\times 36$ and 36×18 respectively) matrices defined by the two (18×18) subblocks corresponding to the identity and the

[12]To make our notations more transparent: each entry in the matrices $\tilde{\mathcal{M}}$ corresponds to a 4×4, 4×2, 2×4 or 2×2 block in chirality space corresponding to the type of coefficient it multiplies (row index) and the one it affects (column index). These blocks involve the 4×4 or 2×2 identities, $\mathbb{1}$, as well as several other matrices, Σ, Ξ, Φ, etc., which we define alongside their first site of appearence. When we mention operators alongside the matrix $\tilde{\mathcal{M}}^{gl}$, in Eq.(18), it is simply in order to clarify which type of operator corresponds to the given row (since we cut the matrix to skip numerous 0 entries).

[13]Note that operators with different quark flavours mix, justifying the use of two quark indices q and q'.

null matrix, when $q = b, s$: their meaning should be clear considering the shortened base in the case $q = b, s$, due to Fierz identities. From our calculation of the divergent contributions, the matrices $\tilde{\mathcal{M}}^{q'q,gq}$ read as:

$$\tilde{\mathcal{M}}^{q'q} = \delta^{q'q}[\mathbb{1};0]_{q'=b,s}\tilde{\mathcal{M}}^{q'q}_{\text{diag}} \begin{bmatrix}\mathbb{1}\\0\end{bmatrix}_{q=b,s} + [\mathbb{1}; -\tilde{V}^{q'}_{\text{Fierz}}]_{q'=b,s}\tilde{\mathcal{M}}^{q'q}_{\text{univ.}} \begin{bmatrix}\mathbb{1}\\-\tilde{V}^{q}_{\text{Fierz}}\end{bmatrix}_{q=b,s} \tag{21}$$

$$\tilde{\mathcal{M}}^{q'q}_{\text{diag}} = \begin{bmatrix}
8\mathbb{1} & 0 & \frac{1}{32}\Delta & 0 & 0 & 0 & 0 & -\frac{3}{32}\Delta & 0 & 0\\
0 & 2\mathbb{1}+\frac{3}{4}\Sigma_3 & 0 & 0 & 0 & 0 & -\frac{9}{4}\Sigma_3 & 0 & 0 & 0\\
24\Delta^T & 0 & 0 & 0 & 0 & -72\Delta^T & 0 & 0 & 0 & 0\\
A_H & B_H & 0 & \mathbb{1} & 0 & C_H & D_H & 0 & 0 & 0\\
A_{\tilde{H}} & B_{\tilde{H}} & 0 & 0 & \mathbb{1} & C_{\tilde{H}} & D_{\tilde{H}} & 0 & 0 & 0\\
\frac{9}{4}\mathbb{1} & 0 & -\frac{3}{64}\Delta & 0 & 0 & \frac{5}{4}\mathbb{1} & 0 & -\frac{7}{64}\Delta & 0 & 0\\
0 & -\frac{9}{8}(\mathbb{1}+\Sigma_3) & 0 & 0 & 0 & 0 & \frac{1}{8}(43\mathbb{1}-21\Sigma_3) & 0 & 0 & 0\\
-36\Delta^T & 0 & -\frac{9}{4}\mathbb{1} & 0 & 0 & -84\Delta^T & 0 & \frac{27}{4}\mathbb{1} & 0 & 0\\
A_{\mathcal{H}} & B_{\mathcal{H}} & 0 & 0 & 0 & C_{\mathcal{H}} & D_{\mathcal{H}} & 0 & \mathbb{1} & 0\\
A_{\tilde{\mathcal{H}}} & B_{\tilde{\mathcal{H}}} & 0 & 0 & 0 & C_{\tilde{\mathcal{H}}} & D_{\tilde{\mathcal{H}}} & 0 & 0 & \mathbb{1}
\end{bmatrix}$$

$$\tilde{\mathcal{M}}^{q'q}_{\text{univ}} = \frac{1}{4}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 & 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 & 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0\\
0 & -\frac{m_q}{3m_b}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 & 0 & \frac{m_q}{m_b}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{cases}
A_H = \frac{m_q}{4m_b}\left[3(\mathbb{1}+\tilde{\Sigma})+\Sigma_3+E\right]\\
A_{\tilde{H}} = \frac{1}{4}\left[3\mathbb{1}+\Sigma_3+\frac{m_s}{m_b}(3\Sigma+\tilde{E})\right]\\
A_{\mathcal{H}} = -\frac{3}{8}\frac{m_q}{m_b}\left[3(\mathbb{1}+\tilde{\Sigma})+\Sigma_3+E\right]\\
A_{\tilde{\mathcal{H}}} = -\frac{3}{8}(3\mathbb{1}+\Sigma_3)-\frac{3}{8}\frac{m_s}{m_b}(3\Sigma+\tilde{E})\\
B_H = \frac{1}{4}\left[-8\mathbb{1}+\Sigma_3+\frac{m_s}{m_b}(-8\Sigma+\tilde{E})\right]\\
B_{\tilde{H}} = \frac{m_q}{4m_b}\left[-8(\mathbb{1}+\tilde{\Sigma})+\Sigma_3+E\right]\\
B_{\mathcal{H}} = -\frac{3}{8}(3\mathbb{1}+\Sigma_3)-\frac{3}{8}\frac{m_s}{m_b}(3\Sigma+\tilde{E})\\
B_{\tilde{\mathcal{H}}} = -\frac{3}{8}\frac{m_q}{m_b}\left[3(\mathbb{1}+\tilde{\Sigma})+\Sigma_3+E\right]\\
C_H = -\frac{3}{4}\frac{m_q}{m_b}\left[3(\mathbb{1}+\tilde{\Sigma})+\Sigma_3+E\right]\\
C_{\tilde{H}} = -\frac{3}{4}\left[(3\mathbb{1}+\Sigma_3)+\frac{m_s}{m_b}(3\Sigma+\tilde{E})\right]\\
C_{\mathcal{H}} = -\frac{7}{8}\frac{m_q}{m_b}\left[3(\mathbb{1}+\tilde{\Sigma})+\Sigma_3+E\right]\\
C_{\tilde{\mathcal{H}}} = -\frac{7}{8}(3\mathbb{1}+\Sigma_3)-\frac{7}{8}\frac{m_s}{m_b}(3\Sigma+\tilde{E})\\
D_H = -\frac{3}{4}(\Sigma_3+\frac{m_s}{m_b}\tilde{E})\\
D_{\tilde{H}} = -\frac{3}{4}\frac{m_q}{m_b}(\Sigma_3+E)\\
D_{\mathcal{H}} = \frac{1}{8}(11\mathbb{1}-7\Sigma_3)+\frac{m_s}{8m_b}(11\Sigma-7\tilde{E})\\
D_{\tilde{\mathcal{H}}} = \frac{m_q}{8m_b}\left[11(\mathbb{1}+\tilde{\Sigma})-7(\Sigma_3+E)\right]
\end{cases} ; \quad
\begin{cases}
\Sigma_3 = \begin{matrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\end{matrix}\\
\\
E = \begin{matrix}0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -1\\ 0 & 0 & 1 & 0\end{matrix}\\
\\
\tilde{E} = \begin{matrix}0 & 0 & 0 & -1\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\end{matrix}\\
\\
\Delta = \begin{matrix}1 & 0\\ 0 & 0\\ 0 & 1\\ 0 & 0\end{matrix}
\end{cases}$$

$$
\tilde{\mathcal{M}}^{gq} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & \cdots & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-12\frac{m_q}{m_b}\Xi & \frac{1}{12}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 & 0 & -\frac{1}{4}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\
24\frac{m_q}{m_b}\tilde{\Xi} & \frac{1}{6}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 & 0 & -\frac{1}{2}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\
-\frac{11}{8}\frac{m_q}{m_b}\Xi & \frac{7}{144}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 & -\frac{15}{8}\frac{m_q}{m_b}\Xi & -\frac{7}{48}(\Xi + \frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\
\frac{11}{4}\frac{m_q}{m_b}\tilde{\Xi} & \frac{7}{72}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 & \frac{15}{4}\frac{m_q}{m_b}\tilde{\Xi} & -\frac{7}{24}(\Xi' + \frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\
0 & 0 & -\frac{3}{64}\frac{m_q}{m_b}\mathbb{1} & 0 & 0 & 0 & 0 & \frac{9}{64}\frac{m_q}{m_b}\mathbb{1} & 0 & 0
\end{bmatrix}
\begin{matrix}
\leftarrow E \\ \cdots \\ \leftarrow \mathcal{H}^T \\ \leftarrow \mathcal{Q}^D \\ \leftarrow \tilde{\mathcal{Q}}^D \\ \leftarrow \mathcal{Q}^S \\ \leftarrow \tilde{\mathcal{Q}}^S \\ \leftarrow \mathcal{Q}^T
\end{matrix}
$$

$$
\times \begin{bmatrix} \mathbb{1} \\ -\tilde{V}^q_{\mathrm{Fierz}} \end{bmatrix}_{q=b,s} \tag{22}
$$

$$
\tilde{\Xi} \equiv \begin{matrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{matrix}
$$

The definition and meaning of the notation $\begin{bmatrix} \mathbb{1} \\ -\tilde{V}^q_{\mathrm{Fierz}} \end{bmatrix}_{q=b,s}$ should be obvious by now.

- $(\bar{b}s)$-Gauge couplings:

$$
\frac{d(C^g)}{d\ln\alpha_S} = -\frac{C_2(3)}{\beta_0}\left\{(C^g)\left[\frac{1}{4}\mathrm{diag}(5\mathbb{1}, 14\mathbb{1}, -19\mathbb{1}, -19\mathbb{1}, -19\mathbb{1}, -10\mathbb{1}, -10\mathbb{1}, -10\mathbb{1}, -\mathbb{1}, -\mathbb{1}, -\mathbb{1}, -\mathbb{1}, -\mathbb{1}) - \tilde{\mathcal{M}}^{gg}\right]\right.
$$

$$
\left. - (C^q)\tilde{\mathcal{M}}^{qg}\right\} + O(\alpha_S) \tag{23}
$$

The matrices $\tilde{\mathcal{M}}^{gg,qg}$ proceed from our calculation of the divergent contributions:

$$
\tilde{\mathcal{M}}^{gg} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-4Q_d\mathbb{1} & \frac{11}{4}\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 4\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{4}\Omega & 0 & -\frac{2}{3}Q_d\mathbb{1} & -\frac{1}{3}Q_d\sigma_3 & 0 & -\frac{55}{24}\mathbb{1} & \frac{11}{48}\sigma_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2}\tilde{\Omega} & 0 & -\frac{4}{3}Q_d\sigma_3 & -\frac{2}{3}Q_d\mathbb{1} & 0 & \frac{11}{12}\sigma_3 & -\frac{55}{24}\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
(1-\frac{m_s^2}{m_b^2})\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}\mathbb{1} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{3}{8}\Omega & 0 & 0 & 0 & -Q_d\mathbb{1} & -\frac{1}{2}Q_d\sigma_3 & 0 & \frac{17}{2}\mathbb{1} & 0 & -\mathbb{1} & -\frac{1}{2}\sigma_3 & \frac{9}{2}\mathbb{1} \\
0 & \frac{3}{4}\tilde{\Omega} & 0 & 0 & 0 & -2Q_d\sigma_3 & -Q_d\mathbb{1} & 0 & 0 & \frac{17}{2}\mathbb{1} & -2\sigma_3 & -\mathbb{1} & -9\sigma_3 \\
0 & -\frac{7}{32}\Omega & 0 & 0 & 0 & -\frac{7}{12}Q_d\mathbb{1} & -\frac{7}{24}Q_d\sigma_3 & 0 & \frac{21}{16}\mathbb{1} & -\frac{3}{32}\sigma_3 & \frac{379}{24}\mathbb{1} & -\frac{41}{48}\sigma_3 & \frac{11}{8}\mathbb{1} \\
0 & \frac{7}{16}\tilde{\Omega} & 0 & 0 & 0 & -\frac{7}{6}Q_d\sigma_3 & -\frac{7}{12}Q_d\mathbb{1} & 0 & -\frac{3}{8}\sigma_3 & \frac{21}{16}\mathbb{1} & -\frac{41}{12}\sigma_3 & \frac{379}{24}\mathbb{1} & -\frac{11}{4}\sigma_3 \\
0 & -\frac{9}{8}(1+\frac{m_s^2}{m_b^2})\mathbb{1} & 0 & 0 & 0 & -\frac{9}{2}Q_d\mathbb{1} & \frac{9}{4}Q_d\sigma_3 & 0 & \frac{3}{2}\mathbb{1} & -\frac{3}{4}\sigma_3 & \frac{45}{2}\mathbb{1} & -\frac{45}{4}\sigma_3 & -\frac{9}{4}\mathbb{1}
\end{bmatrix} \tag{24}
$$

$$
\begin{cases}
\Omega \equiv (1+\frac{m_s^2}{m_b^2})\mathbb{1} - \frac{2}{3}\frac{m_s}{m_b}\sigma_1 \\
\tilde{\Omega} \equiv (1+\frac{m_s^2}{m_b^2})\sigma_3 - \frac{2}{3}\frac{m_s}{m_b}\varepsilon
\end{cases}
\quad ; \quad \sigma_1 \equiv \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \quad ; \quad \sigma_3 \equiv \begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix} \quad ; \quad \varepsilon \equiv \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix}
$$

$$\tilde{\mathcal{M}}^{qg} = [\mathbb{1}; -\tilde{V}^q_{\text{Fierz}}]_{q=b,s} \begin{bmatrix} 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 18Q_q\frac{m_q}{m_b}\mathbb{1} & 0 & 0 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 6Q_q\frac{m_q}{m_b}\mathbb{1} & 6\frac{m_q}{m_b}\mathbb{1} & 0 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{bmatrix} \tag{25}$$

3 Solving the RGE's

Having presented, in the previous section, what is meant to be the main result of this paper, i.e. the RGE's for all operators of mass-dimension ≤ 7 intervening in $b \to s$ transitions, we shall now propose a partial solution for these RGE's, partial in the sense that we will not provide a general solution for operators of dimension 6 but only recover the anomalous-dimension matrix for those operators that are usually considered. On the contrary, for dimension 7 operators, we will present the full result. This section shall also provide us with the opportunity to discuss the limits of our approach and how our extended analysis may be combined with the far-more advanced one of dimension-6 vector operators.

3.1 Dimension 6 operators

First of all, we wish to check whether we can recover the usual leading-order anomalous-dimension matrix for dimension 6 vector operators. We thus consider the basis proposed in Eq.(2.15) of [12]. These can be viewed as specific linear combinations of ours (including also a rescaling), so that we should be able to read the corresponding anomalous-dimension matrix $\gamma_C^{[1-6]}$ from our results. For the time being, we forget about the dimension 7 entries. For the four-quark operators O_{1-6} of [12], we obtain:

$$\gamma_C^{[1-6]} = \frac{g_S^2}{8\pi^2} \begin{bmatrix} -1 & 3 & 0 & 0 & 0 & 0 \\ 3 & -1 & -1/9 & 1/3 & -1/9 & 1/3 \\ 0 & 0 & -11/9 & 11/3 & -2/9 & 2/3 \\ 0 & 0 & 22/9 & 2/3 & -5/9 & 5/3 \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & -5/9 & 5/3 & -5/9 & -19/3 \end{bmatrix} \tag{26}$$

which coincides with the first 6×6 subblock of Eq.(3.4) of this same reference.
Similarly, we consider the so-called electroweak four-quark operators Q_{7-10} of Eq.(VII.2) of [3] and recover the matrix elements presented in Table XIV of this same reference.
Let us turn to the magnetic and chromo-magnetic operators. In our approach, they satisfy the RGE, with $(C_G) \equiv (C_E, C_Q)^{L,R}$:

$$\begin{aligned} \frac{d(C_G)}{d\ln\alpha_S} &= -\frac{C_2(3)}{\beta_0}\left\{(C_G)\begin{bmatrix} \frac{5}{4} & 0 \\ 4Q_d & \frac{3}{4} \end{bmatrix} - (C^q)\tilde{\mathcal{M}}^{qg}\right\} \\ &\simeq -\frac{C_2(3)}{\beta_0}\left\{(C_G)\begin{bmatrix} \frac{5}{4} & 0 \\ -\frac{4}{3} & \frac{3}{4} \end{bmatrix} + (C_S^b, C_T^b)\begin{bmatrix} \frac{1}{16}\Delta & -\frac{3}{16}\Delta \\ 7\mathbb{1} & -3\mathbb{1} \end{bmatrix}\right\} \end{aligned} \tag{27}$$

where we have neglected $\frac{m_q}{m_b} \ll 1$ for $q \neq b$ and replaced $\tilde{V}^b_{\text{Fierz}}$ explicitly. The contributions from the 4-quark operators that we obtain here have no equivalent in the usual approach: the corresponding operators are simply not considered in the classical case, so that it is pointless for us to keep track of those here[14]. Therefore:

$$\begin{cases} C_E(\mu) = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{-\frac{5C_2(3)}{4\beta_0}} C_E(\mu_0) + \frac{8}{3}\left[\left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{-\frac{3C_2(3)}{4\beta_0}} - \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{-\frac{5C_2(3)}{4\beta_0}}\right] C_Q(\mu_0) + (\ldots) \\ C_Q(\mu) = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{-\frac{3C_2(3)}{4\beta_0}} C_Q(\mu_0) + (\ldots) \end{cases} \tag{28}$$

[14] They are symbolically replaced by $(\ldots)$ in Eq.(28). Note that Δ was defined in the previous subsection and appears through $\tilde{V}^q_{\text{Fierz}}$: see Eqs.(21,25).

To recover the usual scaling of the operators $O_{7,8}$ [12], one has to multiply those coefficients by $\frac{m_b}{\alpha_S}$, which leads to the additional factor $\left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{-\frac{11C_2(3)}{4\beta_0}}$. This is, again, consistent with Eq.(3.4) of [12], for the subblock $7-8$.
Yet, one notices at once an apparent discrepancy with Eq.(3.4) of [12]: operators O_{1-6} do not lead to divergences in the $O_{7,8}$ directions. This is not surprising though, in the sense that the corresponding off-diagonal elements appear at the two-loop QCD level while we considered only the one-loop divergences. One may wonder why such two-loop effects in [12] are competitive with the diagonal one-loop elements of the O_{7-8} subblock and, if so, whether it endangers the validity of our one-loop analysis. The explanation is to be found in the relative order of the SM matching for the four-quark and magnetic operators (considering our normalization in Eq.(5,6)): if both were of the same order, then the two-loop mixing would only generate a subleading effect, of relative importance $O(\alpha_S)$, which, for consistency, could be neglected. However, if one refers to the magnitude of the matching elements that we sketch at the end of section 2.1, which is relevant for e.g. the SM, one observes that the matching coefficients of the magnetic operators are already of order α_S (translating the fact that it arises at the loop level even though the α_S factor itself is only an artefact of the relative normalization of the operators), so that the two-loop mixing effect, related to the $O(\alpha_S^0)$ four-quark matching, becomes competitive: in other words, the naive hierarchy has been destabilized by the SM matching. More pragmatically, this situation in the SM is related to the fact that the closure of the charm-quark loop does not entail any α_S suppression. In our naive approach, we would naturally miss such an effect. Nevertheless, one also observes from the estimates at the end of section 2.1 that this unbalance among matching conditions does only concern the dimension 6 four-quark operators O_{1-6}[15], at least in a SM-like model: the corresponding matrix elements are therefore already known and may be included straightforwardly within our analysis. This potentially misleading aspect of our naive approach should, however, be kept in mind in order to combine our findings consistently with the usual dimension 6 analysis, directed at realistic high-energy models.
Let us finally consider the semi-leptonic operators $(C^l) = (C^l_S, C^l_V, C^l_T)$. They satisfy the RGE:

$$\frac{d(C^l)}{d\ln\alpha_S} = \frac{C_2(3)}{\beta_0}\left\{(C^l)\frac{1}{4}\begin{bmatrix} 23\mathbb{1} & 0 & 0 \\ 0 & 11\mathbb{1} & 0 \\ 0 & 0 & 7\mathbb{1}\end{bmatrix} + (C^q)\tilde{\mathcal{M}}_{ql}\right\} \tag{29}$$

leading to:

$$\begin{cases} C^l_S(\mu) = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{23C_2(3)}{4\beta_0}} C^l_S(\mu_0) \\ C^l_V(\mu) = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{11C_2(3)}{4\beta_0}} C^l_V(\mu_0) + (\ldots) \\ C^l_T(\mu) = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{7C_2(3)}{4\beta_0}} C^l_T(\mu_0) \end{cases} \tag{30}$$

$(\ldots)$ stands for the inhomogeneous terms due to four-quark operators which we discuss later. If one factors out α_S^{-1} (for S^l) or $\alpha_S^{-1}m_b$ (for V^l, T^l) in the definition of the operators, the scaling of the coefficients is modified by factors $\left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{-\frac{23C_2(3)}{4\beta_0}}$ and $\left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{-\frac{11C_2(3)}{4\beta_0}}$ (respectively), leading to the commonly known non-running of scalar and vector coefficients: see e.g. [30]. Let us now consider the inhomogeneous terms in the classical case of $O_9 \equiv (\bar{b}\gamma^\mu P_L s)(\bar{l}\gamma_\mu l)$ and $O_{10} \equiv (\bar{b}\gamma^\mu P_L s)(\bar{l}\gamma_\mu\gamma_5 l)$. Then we find that the off-diagonal elements of the anomalous-dimension matrix, translating the mixing with operators O_{1-6}, are given by $\gamma_C^{i7} = \frac{g_S^2}{8\pi^2}\left[-\frac{4}{3}, -\frac{4}{9}, -\frac{2}{9}, \frac{10}{9}, -\frac{2}{3}, -\frac{2}{9}\right]$, $\gamma_C^{i8} = 0$, $i = 1, \ldots, 6$. Up to a factor 2 accounting for the relative normalization of the operators, these matrix elements coincide with Eq.(VIII.11) of [3].
We have thus verified that, except for the mixing of four-quark vector operators with the magnetic and chromo-magnetic operators, which is a two-loop effect, our calculation covered consistently all the leading-order elements of the anomalous-dimension matrix for those operators that had been considered in the literature.
Before turning to the solution for dimension 7 running, it is worthwhile discussing how dimension 7 corrections to these dimension 6 RGE's should be implemented. First note that we are talking here about corrections of order $\frac{m_b}{M_Z}$ at most, so that it would make little sense to include them if the dimension 6 coefficients were considered at leading order only: dimension 7 effects will make sense numerically only if dimension 6 contributions are known up to $O(\alpha_S)$ or even $O(\alpha_S^2)$ (depending on the matching conditions). A second remark comments on the absence of dimension 6 effects in the RGE's of dimension 7 operators (in other words, the anomalous dimension matrix is block-triangular): this naturally proceeds from the fact that the dimension 6 basis is stable and does not

[15] Actually O_2 only, at the level of the matching conditions, but, O_2 closing on the whole O_{1-6} basis under renormalization, the whole subset needs to be included.

require dimension 7 counterterms to cancel its divergences (this can be derived from simple power-counting). The dimension 7 RGE's can therefore be considered separately while their mixing-effect is injected directly as a small inhomogeneous term in the dimension 6 RGE's.

3.2 Dimension 7 RGE's

We present the solution for the RGE's of dimension 7 operators. We stress again that those are independent from the dimension 6 running, allowing us to solve them separately. The block-triangular shape appearing in $\gamma_C^{\text{dim. }7}$, the restriction of the anomalous dimension matrix to dimension 7 operators, invites for a splitting into subblock RGE's with inhomogeneous terms: this observation makes an explicit solution tractable analytically, which we perform for the sake of completeness. Note that the methodology, consisting in splitting the anomalous-dimension matrix according to its block triangular shape and resulting in inhomogeneous linear equations, is a recurring feature in EFT's and has been used, e.g. in [31] (although in a different context; see section 3.4.2 of this reference).

3.2.1 $(\bar{b}s)$-Gauge operators

Gluonic operators; $(C_{\mathcal{Q}}) \equiv (C_{\mathcal{Q}^D}, C_{\tilde{\mathcal{Q}}^D}, C_{\mathcal{Q}^S}, C_{\tilde{\mathcal{Q}}^S}, C_{\mathcal{Q}^T})^{L,R}$:
The RGE is a simple homogeneous 1st-order differential equation:

$$\frac{d(C_{\mathcal{Q}})}{d\ln\alpha_S} = \frac{C_2(3)}{\beta_0}(C_{\mathcal{Q}}) \begin{bmatrix} \frac{35}{4}\mathbb{1} & 0 & -\mathbb{1} & -\frac{1}{2}\sigma_3 & \frac{9}{2}\mathbb{1} \\ 0 & \frac{35}{4}\mathbb{1} & -2\sigma_3 & -\mathbb{1} & -9\sigma_3 \\ \frac{21}{16}\mathbb{1} & -\frac{3}{32}\sigma_3 & \frac{385}{24}\mathbb{1} & -\frac{41}{48}\sigma_3 & \frac{11}{8}\mathbb{1} \\ -\frac{3}{8}\sigma_3 & \frac{21}{16}\mathbb{1} & -\frac{41}{12}\sigma_3 & \frac{385}{24}\mathbb{1} & -\frac{11}{4}\sigma_3 \\ \frac{3}{2}\mathbb{1} & -\frac{3}{4}\sigma_3 & \frac{45}{2}\mathbb{1} & -\frac{45}{4}\sigma_3 & -2\mathbb{1} \end{bmatrix} \equiv \frac{C_2(3)}{\beta_0}(C_{\mathcal{Q}})\mathcal{A}_{\mathcal{Q}\mathcal{Q}}$$

$$\Rightarrow \quad (C_{\mathcal{Q}})(\mu) = (C_Q)(\mu_0)\exp\left[\frac{C_2(3)}{\beta_0}\mathcal{A}_{\mathcal{Q}\mathcal{Q}}\ln\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right] = (C_{\mathcal{Q}})(\mu_0)\mathcal{O}_{\mathcal{Q}}\left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{C_2(3)}{\beta_0}\mathcal{D}_{\mathcal{Q}}}\mathcal{O}_{\mathcal{Q}}^{-1} \quad (31)$$

with $\mathcal{A}_{\mathcal{Q}\mathcal{Q}} \equiv \mathcal{O}_{\mathcal{Q}}\mathcal{D}_{\mathcal{Q}}\mathcal{O}_{\mathcal{Q}}^{-1}$, where $\mathcal{D}_{\mathcal{Q}}$ is a diagonal matrix:
$\mathcal{D}_{\mathcal{Q}} = \text{diag}\left[-\frac{19}{4}\mathbb{1}, \frac{1}{8}(117-\sqrt{3073})\mathbb{1}, \frac{1}{24}(277-\sqrt{3193})\mathbb{1}, \frac{1}{24}(277+\sqrt{3193})\mathbb{1}, \frac{1}{8}(117+\sqrt{3073})\mathbb{1}\right]$
The details of this diagonalization can be found in Appendix C.

Hybrid operators; $(C_{\mathcal{H}}) \equiv (C_{\mathcal{H}^S}, C_{\tilde{\mathcal{H}}^S}, C_{\mathcal{H}^T})^{L,R}$:

$$\frac{d(C_{\mathcal{H}})}{d\ln\alpha_S} = \frac{C_2(3)}{\beta_0}\left\{(C_{\mathcal{H}})\begin{bmatrix} \frac{5}{24}\mathbb{1} & \frac{11}{48}\sigma_3 & 0 \\ \frac{11}{12}\sigma_3 & \frac{5}{24}\mathbb{1} & 0 \\ 0 & 0 & \frac{11}{4}\mathbb{1} \end{bmatrix} - Q_d(C_{\mathcal{Q}})\begin{bmatrix} \mathbb{1} & \frac{1}{2}\sigma_3 & 0 \\ 2\sigma_3 & \mathbb{1} & 0 \\ \frac{7}{12}\mathbb{1} & \frac{7}{24}\sigma_3 & 0 \\ \frac{7}{2}\sigma_3 & \frac{7}{12}\mathbb{1} & 0 \\ \frac{9}{2}\mathbb{1} & -\frac{9}{4}\sigma_3 & 0 \end{bmatrix}\right\}$$

$$\equiv \frac{C_2(3)}{\beta_0}\left\{(C_{\mathcal{H}})\mathcal{A}_{\mathcal{H}\mathcal{H}} - (C_{\mathcal{Q}})\mathcal{A}_{\mathcal{Q}\mathcal{H}}\right\} \quad (32)$$

$$\Rightarrow (C_{\mathcal{H}})(\mu) = \left\{(C_{\mathcal{H}})(\mu_0) - \frac{C_2(3)}{\beta_0}\int_0^{\ln\left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)}(C_{\mathcal{Q}})(\mu')\mathcal{A}_{\mathcal{Q}\mathcal{H}}\exp\left[-\frac{C_2(3)}{\beta_0}\mathcal{A}_{\mathcal{H}\mathcal{H}}\ln\frac{\alpha_S(\mu')}{\alpha_S(\mu_0)}\right]d\ln\left(\frac{\alpha_S(\mu')}{\alpha_S(\mu_0)}\right)\right\}$$

$$\times\exp\left[\frac{C_2(3)}{\beta_0}\mathcal{A}_{\mathcal{H}\mathcal{H}}\ln\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right] \quad (33)$$

The exponentiation of $\mathcal{A}_{\mathcal{H}\mathcal{H}}$ can be achieved without difficulty through its diagonalization: $\mathcal{A}_{\mathcal{H}\mathcal{H}} = \mathcal{O}_{\mathcal{H}}\mathcal{D}_{\mathcal{H}}\mathcal{O}_{\mathcal{H}}^{-1}$, with $\mathcal{D}_{\mathcal{H}} \equiv \text{diag}\left[-\frac{1}{4}\mathbb{1}, \frac{2}{3}\mathbb{1}, \frac{11}{4}\mathbb{1}\right]$ (refer to Appendix C).

Photonic operators; $(C_{\mathcal{E}}) \equiv (C_{\mathcal{E}^S}, C_{\tilde{\mathcal{E}}^S}, C_{\mathcal{E}^T})^{L,R}$:

$$\frac{d(C_{\mathcal{E}})}{d\ln\alpha_S} = \frac{C_2(3)}{\beta_0}\left\{(C_{\mathcal{E}})\frac{1}{4}\begin{bmatrix} 35\mathbb{1} & 0 & 0 \\ 0 & 35\mathbb{1} & 0 \\ 0 & 0 & 19\mathbb{1} \end{bmatrix} - (C_{\mathcal{H}})Q_d\begin{bmatrix} \frac{2}{3}\mathbb{1} & \frac{1}{3}\sigma_3 & 0 \\ \frac{4}{3}\sigma_3 & \frac{2}{3}\mathbb{1} & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\} \quad (34)$$

$$\Rightarrow (C_{\mathcal{E}})(\mu) = (C_{\mathcal{E}})(\mu_0) \begin{bmatrix} \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{35C_2(3)}{4\beta_0}} \mathbb{1} & 0 & 0 \\ 0 & \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{35C_2(3)}{4\beta_0}} \mathbb{1} & 0 \\ 0 & 0 & \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{19C_2(3)}{4\beta_0}} \mathbb{1} \end{bmatrix}$$
$$- \frac{C_2(3)Q_d}{\beta_0} \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{35C_2(3)}{4\beta_0}} \int_0^{\ln\left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)} (C_{\mathcal{H}})(\mu') \left(\frac{\alpha_S(\mu')}{\alpha_S(\mu_0)}\right)^{-\frac{35C_2(3)}{4\beta_0}} d\ln\left(\frac{\alpha_S(\mu')}{\alpha_S(\mu_0)}\right) \begin{bmatrix} \frac{2}{3}\mathbb{1} & \frac{1}{3}\sigma_3 & 0 \\ \frac{4}{3}\sigma_3 & \frac{2}{3}\mathbb{1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (35)$$

Interestingly, certain of those operators have a negative anomalous dimension, meaning that they will grow at low energy, and thus become important. Note however that this negative dimension may simply be an artefact of artificial factors α_S^{-1} in the normalization of the operators: the matching conditions in a definite model are therefore likely to correct this feature by introducing suppression factors. Otherwise, such growing directions in the dimension 7 anomalous-dimension matrix could lead to significant perturbation of the dimension 6 results. While the computation of the anomalous dimension is particularly simple, e.g. in the case of the photonic operators (leaving little room for false moves), the prospect of dimension 7 contributions enhanced at low energy should be considered however with caution.

3.2.2 Dimension 7 $(\bar{b}s)(\bar{q}q)$ operators

$(C_H^q) \equiv (C^q_{H^q}, C^q_{\tilde{H}^q}, C^q_{\mathcal{H}^q}, C^q_{\tilde{\mathcal{H}}^q})^{L,R\,L,R}$:

$$\frac{d(C_H^q)}{d\ln\alpha_S} = -\frac{C_2(3)}{\beta_0} \left\{ (C_H^q) - (C_H^{q'}) \frac{1}{4} [\mathbb{1}; -\tilde{V}^{q'}_{\text{Fierz}}]_{q'=b,s} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbb{1} \\ -\tilde{V}^q_{\text{Fierz}} \end{bmatrix}_{q=b,s} \right\}$$
$$\equiv -\frac{C_2(3)}{\beta_0} (C_H^q) \left[\mathbb{1} + \mathcal{A}_{qq}\right] \quad (36)$$

$$\Rightarrow (C_H^q)(\mu) = (C_H^q)(\mu_0) \exp\left[-\frac{C_2(3)}{\beta_0} (\mathbb{1}+\mathcal{A}_{qq}) \ln\frac{\alpha_S(\mu')}{\alpha_S(\mu_0)}\right]$$

$\mathcal{A}_{qq}$ can be explicitly diagonalized: $\mathcal{A}_{qq} = \mathcal{O}_q \text{diag}(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{8}{3}\mathcal{D}_{24})\mathcal{O}_q^{-1}$ (refer to Appendix C).

3.2.3 Dimension 7 $(\bar{b}s)(\bar{l}l)$ operators

$(C_H^l) \equiv (C^l_{H^l}, C^l_{\tilde{H}^l})$:

$$\frac{d(C_H^l)}{d\ln\alpha_S} = \frac{C_2(3)}{4\beta_0} (C_H^l) \begin{bmatrix} 19\mathbb{1} & 0 \\ 0 & 23\mathbb{1} \end{bmatrix} - \frac{2Q_lQ_q}{\beta_0} (C_H^q) [\mathbb{1}; -\tilde{V}^q_{\text{Fierz}}]_{q=b,s} \begin{bmatrix} \mathbb{1}+\tilde{\Sigma} & 0 \\ 0 & 0 \\ \frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 \\ 0 & 0 \end{bmatrix} \quad (37)$$

$$\Rightarrow (C_H^l)(\mu) = (C_H^l)(\mu_0) \begin{bmatrix} \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{19C_2(3)}{4\beta_0}} \mathbb{1} & 0 \\ 0 & \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{23C_2(3)}{4\beta_0}} \mathbb{1} \end{bmatrix}$$
$$- \frac{2Q_lQ_q}{\beta_0} \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{19C_2(3)}{4\beta_0}} \int_0^{\ln\left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)} (C_H^q)(\mu') \left(\frac{\alpha_S(\mu')}{\alpha_S(\mu_0)}\right)^{-\frac{19C_2(3)}{4\beta_0}} d\ln\left(\frac{\alpha_S(\mu')}{\alpha_S(\mu_0)}\right)$$
$$\times [\mathbb{1}; -\tilde{V}^q_{\text{Fierz}}]_{q=b,s} \begin{bmatrix} \mathbb{1}+\tilde{\Sigma} & 0 \\ 0 & 0 \\ \frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 \\ 0 & 0 \end{bmatrix} \quad (38)$$

Note that the positive eigenvalues (negative anomalous dimensions) change sign once the artificial factor α_S^{-1} is factored away from the operators.
This concludes the solution of all dimension 7 RGE's. We will now consider a simple example engaging these dimension 7 effects.

4 A simple application

4.1 Setup

Let us assume that leading new-physics effects would arise in the possible (though unlikely) form of the following dimension 6 SM operator [27]:

$$\mathcal{L}_{\text{NP}} = -\frac{\imath K_{sb}}{M_Z^2} H^\dagger \frac{g' B_{\mu\nu} + 2g W^a_{\mu\nu}\tau^a}{\sqrt{g^2+g'^2}} \bar{s}\sigma^{\mu\nu} P_L \begin{pmatrix} t \\ b \end{pmatrix} + h.c. \tag{39}$$

where g' and g are the $U(1)_Y$ and $SU(2)_L$ coupling constants, $B_{\mu\nu}$ and $W^a_{\mu\nu}\tau^a$ the corresponding field-strength tensors (τ^a correspond to the $SU(2)_L$ generators), while H is the Higgs doublet (with negative hypercharge). K_{sb} is a complex coefficient encoding new-physics effects. At low-energy, it results in a modified $Z-b-s$ coupling:

$$\frac{\imath K^*_{sb} v}{M_Z^2}\left[\bar{b}\sigma^{\mu\nu}P_R s\right](\partial_\mu Z_\nu - \partial_\nu Z_\mu) + h.c. \tag{40}$$

where $v = (2\sqrt{2}G_F)^{-1/2}$ is the electroweak vacuum expectation value. Such a term could generate flavour effects at the Z-pole, resulting in a perturbation of the precision measurements ($Z \to \bar{b}s, \bar{s}b$ being mistaken experimentally for a $Z \to \bar{b}b$ decay).
Considering the modified Z-coupling, we compute the associated matching conditions, at tree-level, for the operators of the $b \to s$ transition:

$$\begin{cases} \delta^{\text{NP}} C^{L\,L}_{V^f}(M_Z) = -4\sqrt{2}(I_3^f - Q^f s_W^2)\frac{m_s K^*_{sb}}{M_Z^3} \simeq 0 \\ \delta^{\text{NP}} C^{L\,R}_{V^f}(M_Z) = 4\sqrt{2}Q^f s_W^2 \frac{m_s K^*_{sb}}{M_Z^3} \simeq 0 \\ \delta^{\text{NP}} C^{R\,R}_{V^f}(M_Z) = -4\sqrt{2}Q^f s_W^2 \frac{m_b(M_Z) K^*_{sb}}{M_Z^3} \\ \delta^{\text{NP}} C^{R\,L}_{V^f}(M_Z) = 4\sqrt{2}(I_3^f - Q^f s_W^2)\frac{m_b(M_Z) K^*_{sb}}{M_Z^3} \end{cases} ; \quad \begin{cases} \delta^{\text{NP}} C^{L\,L}_{H^f}(M_Z) = 0 \\ \delta^{\text{NP}} C^{L\,R}_{H^f}(M_Z) = 0 \\ \delta^{\text{NP}} C^{R\,R}_{H^f}(M_Z) = 4\sqrt{2}Q^f s_W^2 \frac{K^*_{sb}}{M_Z^3} \\ \delta^{\text{NP}} C^{R\,L}_{H^f}(M_Z) = -4\sqrt{2}(I_3^f - Q^f s_W^2)\frac{K^*_{sb}}{M_Z^3} \end{cases} \tag{41}$$

where we here use the following normalization of the operators (f stands for any of the low-energy quarks and leptons):

$$(V^f)^{L,R\;L,R} = (\bar{b}\gamma^\mu P_{L,R}s)(\bar{f}\gamma_\mu P_{L,R}f) \quad ; \quad (H^f)^{L,R\;L,R} = [\bar{b}\imath(\overrightarrow{D} - \overleftarrow{D})^\mu P_{L,R}s](\bar{f}\gamma_\mu P_{L,R}f) \tag{42}$$

Note, at this point, that the classical framework of dimension 6 vector-type operators would not allow for a consistent description of these new-physics effects: albeit the $(V^f)^{L,R\;L,R}$ operators receive a contribution, the associated effect is of the same order as that of the dimension 7 $(H^f)^{L,R\;L,R}$ operators. One should thus rely on our extended analysis.

4.2 $BR(B_s \to l^+l^-)$

Now let us consider the decay $B_s \to l^+l^-$. This rate is one of the traditional search channels for new-physics. Evidence for its observation (in the case $l = \mu$) has been reported a few months ago at LHCb [1], evidencing a good agreement with the SM, hence constraining new-physics effects more tightly. For a recent review, refer to [2]. The well-known SM-matching provides [3] (the function Y is defined in this reference):

$$C^{L\,L}_{V^l}(M_Z) \simeq -\frac{\sqrt{2}G_F\alpha}{\pi s_W^2} V^*_{ts}V_{tb} Y(\frac{m_t^2}{M_W^2}) \simeq -6.2 \cdot 10^{-9} \equiv C^{\text{SM}}_{V^l} \tag{43}$$

We (safely) neglect dimension 7 effects associated with the SM and assume that only new physics associated with K_{sb} may lead to a significant deviation.

Then, we consider the following matrix elements (in terms of the B_s-meson decay constant f_B, and its four-momentum P_μ):

$$\begin{aligned}
&\langle 0|\,\bar b\gamma_5 s\,|B_s\rangle = -\imath f_B\frac{M_B^2}{m_b+m_s} && \langle 0|\,\bar b s\,|B_s\rangle = 0\\
&\langle 0|\,\bar b\gamma_\mu\gamma_5 s\,|B_s\rangle = \imath f_B P_\mu && \langle 0|\,\bar b\gamma_\mu s\,|B_s\rangle = 0\\
&\langle 0|\,\bar b\imath(\overrightarrow{D}-\overleftarrow{D})_\mu\gamma_5 s\,|B_s\rangle = \imath f_B(m_b-m_s)P_\mu && \langle 0|\,\bar b\imath(\overrightarrow{D}-\overleftarrow{D})_\mu s\,|B_s\rangle = 0
\end{aligned} \tag{44}$$

Here we have followed the conventions of [32] and derived the result for the matrix elements $\langle 0|\,\bar b\imath(\overrightarrow{D}-\overleftarrow{D})_\mu\mathbb{1}/\gamma_5 s\,|B_s\rangle$. One then derives (where we consider only the operators V^l and H^l, relevant in this particular example):

$$BR(B_s\to l^+l^-)\simeq\frac{f_B^2M_Bm_l^2}{32\pi\Gamma_B}\sqrt{1-4\frac{m_l^2}{M_B^2}}\left|C_{V^l}^{L\,R}-C_{V^l}^{L\,L}+C_{V^l}^{R\,L}-C_{V^l}^{R\,R}+m_b\left(C_{H^l}^{L\,R}-C_{H^l}^{L\,L}+C_{H^l}^{R\,L}-C_{H^l}^{R\,R}\right)\right|^2(\mu_b) \tag{45}$$

Note that for the combinations $C_{V^l}^{L,R\,L}-C_{V^l}^{L,R\,R}$ and $C_{H^l}^{L,R\,L}-C_{H^l}^{L,R\,R}$, the mixing to the four-quark operators cancels in the RGE (Eq.(16), due to the $\mathbb{1}+\tilde\Sigma$ in $\tilde{\mathcal{M}}^{ql}$) so that we obtain simple scalings for both these quantities: $\left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)}\right)^0=1$ for $C_{V^l}^{L,R\,L}-C_{V^l}^{L,R\,R}$ and $\left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)}\right)^{-\frac{C_2(3)}{\beta_0}}=\left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)}\right)^{-\frac{4}{23}}$ for $C_{H^l}^{L,R\,L}-C_{H^l}^{L,R\,R}$. Note however the off-diagonal term in $\tilde{\mathcal{M}}^{ll}$ mixing V^l and H^l operators. The RGE for the vector coefficients hence reads:

$$\begin{aligned}
&\frac{d}{d\ln\alpha_S}[C_{V^l}^{L,R\,L}-C_{V^l}^{L,R\,R}]=-\frac{2C_2(3)}{\beta_0}m_b[C_{H^l}^{L,R\,L}-C_{H^l}^{L,R\,R}]\quad\Rightarrow\\
&[C_{V^l}^{L,R\,L}-C_{V^l}^{L,R\,R}](\mu_b)=[C_{V^l}^{L,R\,L}-C_{V^l}^{L,R\,R}](M_Z)-\frac{2C_2(3)}{\beta_0}m_b(M_Z)[C_{H^l}^{L,R\,L}-C_{H^l}^{L,R\,R}](M_Z)\int_0^{\ln\left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)}\right)}e^{\frac{2C_2(3)}{\beta_0}x}\,dx\\
&\qquad=[C_{V^l}^{L,R\,L}-C_{V^l}^{L,R\,R}](M_Z)-m_b(M_Z)[C_{H^l}^{L,R\,L}-C_{H^l}^{L,R\,R}](M_Z)\left[\left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)}\right)^{\frac{2C_2(3)}{\beta_0}}-1\right]
\end{aligned} \tag{46}$$

It is also more convenient to work with a low-energy b mass: $m_b=m_b(M_Z)\left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)}\right)^{\frac{3C_2(3)}{\beta_0}}$. Note that this replacement does not concern the explicit factor m_b multiplying the H^l coefficients in Eq.(45). Replacing the coefficients at the M_Z scale by the matching conditions, we then observe a complete cancellation of the new-physics contributions:

$$BR(B_s\to l^+l^-)\simeq\frac{f_B^2M_Bm_l^2}{32\pi\Gamma_B}\sqrt{1-4\frac{m_l^2}{M_B^2}}\,|C_{V^l}^{\rm SM}|^2 \tag{47}$$

The scenario under investigation hence receives no constraint from $BR(B_s\to l^+l^-)$. Although this is what the naive tree-level calculation would have predicted, it is a non-trivial result to observe that this feature is preserved by the resummation of the leading logarithms through the RGE's: inconsistently neglecting the H contributions or the $V-H$ mixing, for instance, would have generated limits of the order $|K_{sb}|\lesssim 10^{-3}$. Note however that the cancellation of the new-physics effects is tightly related to the form of the matching conditions of Eq.(41) and that a perturbation of Eq.(41), e.g. through the implementation of additional dimension 6 operators, would lead to relevant limits again.

4.3 $BR(B\to K\nu\bar\nu)$

Many other $b\to s$ transitions are of course relevant to constrain K_{sb}. Let us consider another simple example: the decay $B\to K\nu\bar\nu$ is bounded by the limit $BR(B^-\to K^-+\text{Inv.})<13\cdot10^{-6}$ (90% C.L.) from BABAR [33]. The SM provides $BR(B^-\to K^-\nu\bar\nu)\simeq(4.4^{+1.3+0.8+0.0}_{-1.1-0.7-0.7})\cdot10^{-6}$: refer to [34] for a summary. The relevant matching coefficient for the SM can be read in [3] (the function X is defined in this reference):

$$C_{V^\nu}^{L\,L}(M_Z)\simeq\frac{\sqrt2 G_F\alpha}{\pi s_W^2}V_{ts}^*V_{tb}X(\frac{m_t^2}{M_W^2})\simeq 1\cdot10^{-8}\equiv C_{V^\nu}^{\rm SM} \tag{48}$$

Concerning the decay constants, we again follow the conventions of [32] and derive the matrix element corresponding to the H-type operators:

$$\begin{aligned}
&< K(P_K)|\bar{s}\gamma_\mu b|B(P_B) >= [P_B+P_K]_\mu f_+(s) + \frac{M_B^2-M_K^2}{s}[P_B-P_K]_\mu(f_0(s)-f_+(s)) \quad s\equiv (P_B-P_K)^2 \\
&< K(P_K)|\bar{s}\frac{\imath}{2}\sigma_{\mu\nu}(P_B-P_K)^\nu b|B(P_B) >= \imath\left\{[P_B+P_K]_\mu\frac{s}{M_B+M_K} - [P_B-P_K]_\mu(M_B-M_K)\right\} f_T(s) \\
&< K(P_K)|\bar{s}\imath(\overrightarrow{D}-\overleftarrow{D})_\mu b|B(P_B) >\simeq [P_B+P_K]_\mu\left\{m_b f_+(s) - \frac{sf_T(s)}{M_B+M_K}\right\} \\
&\qquad + [P_B-P_K]_\mu\left\{m_b\frac{M_B^2-M_K^2}{s}(f_0(s)-f_+(s)) + (M_B-M_K)f_T(s)\right\}
\end{aligned} \tag{49}$$

For the form factors, we follow the discussion in [34] closely, employing the parametrization of [35] and the approximate relation $f_T(s)/f_+(s)\simeq\frac{M_B+M_K}{M_B}$.
The estimation of the branching ratio is now straightforward ($N_\nu=3$ is the number of neutrino flavours):

$$\begin{aligned}
BR(B\to K\nu\bar{\nu}) = &\frac{N_\nu/3}{512\pi^3M_B^3\Gamma_B}\int_0^{(M_B-M_K)^2} ds\left[s^2-2s(M_B^2+M_K^2)+(M_B^2-M_K^2)^2\right]^{3/2} \\
&\times\left|[C_{V^\nu}^{L\,L}+C_{V^\nu}^{R\,L}+m_b(C_{H^\nu}^{L\,L}+C_{H^\nu}^{R\,L})] - \frac{sf_T(s)}{M_B+M_K}(C_{H^\nu}^{L\,L}+C_{H^\nu}^{R\,L})\right|^2(\mu_b)
\end{aligned} \tag{50}$$

where we will use $M_B=5.27925(17)$ GeV, $\tau_B=(1.641\pm0.008)\cdot10^{-12}$ s, $M_K=0.493677(16)$ GeV [36].
The running of the coefficients is again very simple as the neutrality of the neutrinos ensures no mixing with the four-quark operators. Similarly to Eq.(46), we have:

$$\begin{aligned}
&[C_{V^\nu}^{L\,L}+C_{V^\nu}^{R\,L}](\mu_b) = [C_{V^\nu}^{L\,L}+C_{V^\nu}^{R\,L}](M_Z) - m_b(M_Z)[C_{H^\nu}^{L\,L}+C_{H^\nu}^{R\,L}](M_Z)\left[\left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)}\right)^{\frac{2C_2(3)}{\beta_0}}-1\right] \\
&[C_{H^\nu}^{L\,L}+C_{H^\nu}^{R\,L}](\mu_b) = \left(\frac{\alpha_S(\mu_b)}{\alpha_S(M_Z)}\right)^{-\frac{C_2(3)}{\beta_0}}[C_{H^\nu}^{L\,L}+C_{H^\nu}^{R\,L}](M_Z)
\end{aligned} \tag{51}$$

Going to our explicit matching conditions of Eq.(41), we observe that new-physics effects here persist in the part multiplying $f_T(s)$. Numerically, we find K_{sb} (which we assume to be real) in the range $[-4,1.5]\cdot10^{-3}$. The naive estimate $K_{sb}\sim10^{-3}\sim\frac{M_Z^2}{\Lambda_{NP}^2}$ returns $\Lambda_{NP}\sim$ TeV; this scale can be lowered further if one assumes that the new-physics operators should be loop-suppressed, follow Minimal Flavour Violation, etc. The bound on new physics appearing in this fashion is therefore relatively loose. Note however that several other observables in the $b\to s$ sector should be considered before drawing any conclusion concerning the viability of $K_{sb}\sim10^{-3}$. Since our focus in this section was merely to consider a concrete, yet simple, case where the inclusion of the dimension 7 RGE's was relevant, we shall not pursue this analysis further.

Let us briefly summarize our achievements. we have established the most general basis of (on-shell) operators, for the EFT describing $b\to s$ transitions, up to mass-dimension 7. After computing the associated ultraviolet QCD divergences at leading order, we have derived the corresponding RGE's. Comparison with the existing studies concerning dimension 6 operators proved satisfactory. We have also solved all the RGE's describing the evolution of pure dimension 7 operators: interestingly, we observed that some directions exhibited negative anomalous dimensions, which could lead to an enhancement effect at low energy. Note however that this property depends on the normalization of the operators so that the matching conditions in a definite high-energy model are likely to regulate it. We finally used this analysis to constrain, using the measurement of the decay $B_s^0\to\mu^+\mu^-$ at LHCb and the limit set by BABAR on $B\to K\nu\bar{\nu}$, a very naive extension of the SM resulting from the addition of a dimension 6 SM-operator. More generally, let us recall that the inclusion of dimension 7 effects has little relevance in e.g. the SM, where such operators receive extra-suppression due to the very-constrained pattern of flavour-violation. Beyond the SM, requiring New-Physics to project only on operators of higher-mass dimension would be an elegant way to circumvent the strong limits on non-standard flavour violation: however, such models would have to be designed on purpose.

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A Fierz identities

We derive/recover the following Fierz identities[16]:

- Scalar identities:

$$\begin{cases} (\bar{\psi}' P_L \psi)(\bar{\chi}' P_L \chi) = \frac{1}{2}(\bar{\psi}' P_L \chi)(\bar{\chi}' P_L \psi) - \frac{1}{32}(\bar{\psi}' \sigma^{\mu\nu} P_L \chi)(\bar{\chi}' \sigma_{\mu\nu} P_L \psi) \\ (\bar{\psi}' P_R \psi)(\bar{\chi}' P_R \chi) = \frac{1}{2}(\bar{\psi}' P_R \chi)(\bar{\chi}' P_R \psi) - \frac{1}{32}(\bar{\psi}' \sigma^{\mu\nu} P_R \chi)(\bar{\chi}' \sigma_{\mu\nu} P_R \psi) \\ (\bar{\psi}' P_L \psi)(\bar{\chi}' P_R \chi) = \frac{1}{2}(\bar{\psi}' \gamma^\mu P_R \chi)(\bar{\chi}' \gamma_\mu P_L \psi) \\ (\bar{\psi}' P_R \psi)(\bar{\chi}' P_L \chi) = \frac{1}{2}(\bar{\psi}' \gamma^\mu P_L \chi)(\bar{\chi}' \gamma_\mu P_R \psi) \end{cases} \tag{52}$$

- Vector identities:

$$\begin{cases} (\bar{\psi}' \gamma^\mu P_L \psi)(\bar{\chi}' \gamma_\mu P_L \chi) = -(\bar{\psi}' \gamma^\mu P_L \chi)(\bar{\chi}' \gamma_\mu P_L \psi) \\ (\bar{\psi}' \gamma^\mu P_R \psi)(\bar{\chi}' \gamma_\mu P_R \chi) = -(\bar{\psi}' \gamma^\mu P_R \chi)(\bar{\chi}' \gamma_\mu P_R \psi) \\ (\bar{\psi}' \gamma^\mu P_L \psi)(\bar{\chi}' \gamma_\mu P_R \chi) = 2(\bar{\psi}' P_R \chi)(\bar{\chi}' P_L \psi) \\ (\bar{\psi}' \gamma^\mu P_R \psi)(\bar{\chi}' \gamma_\mu P_L \chi) = 2(\bar{\psi}' P_L \chi)(\bar{\chi}' P_R \psi) \end{cases} \tag{53}$$

- Tensor identities:

$$\begin{cases} (\bar{\psi}' \sigma^{\mu\nu} P_L \psi)(\bar{\chi}' \sigma_{\mu\nu} P_L \chi) = -24(\bar{\psi}' P_L \chi)(\bar{\chi}' P_L \psi) - \frac{1}{2}(\bar{\psi}' \sigma^{\mu\nu} P_L \chi)(\bar{\chi}' \sigma_{\mu\nu} P_L \psi) \\ (\bar{\psi}' \sigma^{\mu\nu} P_R \psi)(\bar{\chi}' \sigma_{\mu\nu} P_R \chi) = -24(\bar{\psi}' P_R \chi)(\bar{\chi}' P_R \psi) - \frac{1}{2}(\bar{\psi}' \sigma^{\mu\nu} P_R \chi)(\bar{\chi}' \sigma_{\mu\nu} P_R \psi) \end{cases} \tag{54}$$

- Hybrid identities:

$$\begin{cases} (\bar{\psi}' P_L \imath D^\mu \psi)(\bar{\chi}' \gamma_\mu P_L \chi) = \frac{1}{2}(\bar{\psi}' P_L \chi)(\bar{\chi}' P_R \imath \not{D} \psi) - \dfrac{1}{4}(\bar{\psi}' \sigma^{\mu\nu} P_L \chi)(\bar{\chi}' \gamma_\nu P_L \imath D_\mu \psi) \\ (\bar{\psi}' P_R \imath D^\mu \psi)(\bar{\chi}' \gamma_\mu P_R \chi) = \frac{1}{2}(\bar{\psi}' P_R \chi)(\bar{\chi}' P_L \imath \not{D} \psi) - \dfrac{1}{4}(\bar{\psi}' \sigma^{\mu\nu} P_R \chi)(\bar{\chi}' \gamma_\nu P_R \imath D_\mu \psi) \\ (\bar{\psi}' P_L \imath D^\mu \psi)(\bar{\chi}' \gamma_\mu P_R \chi) = (\bar{\psi}' \gamma^\mu P_R \chi)(\bar{\chi}' P_L \imath D_\mu \psi) - \frac{1}{2}(\bar{\psi}' \gamma^\mu P_R \chi)(\bar{\chi}' \gamma_\mu P_R \imath \not{D} \psi) \\ (\bar{\psi}' P_R \imath D^\mu \psi)(\bar{\chi}' \gamma_\mu P_L \chi) = (\bar{\psi}' \gamma^\mu P_L \chi)(\bar{\chi}' P_R \imath D_\mu \psi) - \frac{1}{2}(\bar{\psi}' \gamma^\mu P_L \chi)(\bar{\chi}' \gamma_\mu P_L \imath \not{D} \psi) \end{cases} \tag{55}$$

One can straightforwardly generalize these results to the case where the covariant derivative acts on the first spinor, instead of the second. Moreover, with a little bit of algebra:

$$\begin{aligned} (\bar{\psi}' \sigma^{\mu\nu} P_{L,R} \chi)(\bar{\chi}' \gamma_\nu P_{L,R} \imath D_\mu \psi) = &-[\bar{\psi}' \imath(\overrightarrow{D} - \imath \overleftarrow{D})_\mu P_{L,R} \chi](\bar{\chi}' \gamma^\mu P_{L,R} \psi) - [\bar{\psi}' \imath(\overrightarrow{D} - \imath \overleftarrow{D})_\mu \gamma_5 P_{L,R} \chi](\bar{\chi}' \gamma^\mu \gamma_5 P_{L,R} \psi) \\ &- [\bar{\psi}'(\imath \overleftarrow{\not{D}} \gamma^\mu P_{L,R} - \gamma^\mu P_{R,L} \imath \overrightarrow{\not{D}})\chi](\bar{\chi}' \gamma_\mu P_{L,R} \psi) - [\bar{\psi}'(\imath \overleftarrow{\not{D}} \gamma^\mu \gamma_5 P_{L,R} + \gamma^\mu \gamma_5 P_{R,L} \imath \overrightarrow{\not{D}})\chi](\bar{\chi}' \gamma_\mu \gamma_5 P_{L,R} \psi) \\ &- \frac{1}{4}(\bar{\psi}' \sigma^{\mu\nu} P_{L,R} \chi)[\bar{\chi}'(\overleftarrow{\not{D}} \sigma_{\mu\nu} P_{L,R} + \sigma_{\mu\nu} P_{R,L} \overrightarrow{\not{D}})\psi] - \frac{1}{4} \imath \partial_\rho [(\bar{\psi}' \sigma_{\mu\nu} P_{L,R} \chi)(\bar{\chi}' \gamma^\rho \gamma^\nu) \gamma^\mu P_{L,R} \psi] \end{aligned} \tag{56}$$

Consequently, for $q = b, s$, $\mathcal{O}^q_{L,R\ L,R} = \mathcal{S}, \mathcal{V}, \mathcal{T}, \mathcal{H}, \tilde{\mathcal{H}}^q_{L,R\ L,R}$ can be expressed in terms of $O^q_{L,R\ L,R} = S, V, T, H$, $\tilde{H}^q_{L,R\ L,R}$ as $\mathcal{O}^q_{L,R\ L,R} = -\tilde{V}^q_{\text{Fierz}} O^q_{L,R\ L,R}$ (the sign originates from the necessity of anticommuting fermion fields),

[16]Note that, here, we consider the identities only from the point of view of the Clifford algebra, without including the − sign resulting from the anticommutation of fermion fields.

and reciprocally. Chiralities are ordered as LL, LR, RR, RL.

$$\tilde{V}^b_{\mathrm{Fierz}} = \begin{bmatrix}
\begin{matrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{matrix} & \begin{matrix} -\frac{1}{32} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{32} \\ 0 & 0 \end{matrix} & 0 & 0 \\
\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{matrix} & \begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & 0 & 0 & 0 \\
\begin{matrix} -24 & 0 & 0 & 0 \\ 0 & 0 & -24 & 0 \end{matrix} & 0 & \begin{matrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{matrix} & 0 & 0 \\
\begin{matrix} -\frac{1}{2} & 0 & 0 & \frac{m_s}{m_b} \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & \frac{m_s}{m_b} & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \end{matrix} & \begin{matrix} -1 & -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{m_s}{m_b} & 0 \\ 0 & 0 & -1 & -\frac{1}{2} \\ -\frac{m_s}{m_b} & 0 & 0 & -\frac{1}{2} \end{matrix} & \begin{matrix} -\frac{1}{32} & 0 \\ -\frac{1}{32} & 0 \\ 0 & -\frac{1}{32} \\ 0 & -\frac{1}{32} \end{matrix} & \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} & \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \\
\begin{matrix} -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & -\frac{m_s}{2m_b} & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \\ -\frac{m_s}{2m_b} & 0 & 0 & 1 \end{matrix} & \begin{matrix} -1 & 0 & 0 & -\frac{m_s}{2m_b} \\ -1 & -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{m_s}{2m_b} & -1 & 0 \\ 0 & 0 & -1 & -\frac{1}{2} \end{matrix} & \begin{matrix} -\frac{1}{32} & 0 \\ 0 & -\frac{m_s}{32m_b} \\ 0 & -\frac{1}{32} \\ -\frac{m_s}{32m_b} & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}
\end{bmatrix} \tag{57}$$

$$\tilde{V}^s_{\mathrm{Fierz}} = \begin{bmatrix}
\begin{matrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \end{matrix} & \begin{matrix} -\frac{1}{32} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{32} \\ 0 & 0 \end{matrix} & 0 & 0 \\
\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{matrix} & \begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & 0 & 0 & 0 \\
\begin{matrix} -24 & 0 & 0 & 0 \\ 0 & 0 & -24 & 0 \end{matrix} & 0 & \begin{matrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{matrix} & 0 & 0 \\
\begin{matrix} -\frac{m_s}{2m_b} & \frac{m_s}{m_b} & 0 & 0 \\ -\frac{m_s}{2m_b} & 0 & 0 & 1 \\ 0 & 0 & -\frac{m_s}{2m_b} & \frac{m_s}{m_b} \\ 0 & 1 & -\frac{m_s}{2m_b} & 0 \end{matrix} & \begin{matrix} -1 & 0 & 0 & -\frac{m_s}{2m_b} \\ 0 & 0 & -\frac{m_s}{m_b} & -\frac{m_s}{2m_b} \\ 0 & -\frac{m_s}{2m_b} & -1 & 0 \\ -\frac{m_s}{m_b} & -\frac{m_s}{2m_b} & 0 & 0 \end{matrix} & \begin{matrix} -\frac{m_s}{32m_b} & 0 \\ -\frac{m_s}{32m_b} & 0 \\ 0 & -\frac{m_s}{32m_b} \\ 0 & -\frac{m_s}{32m_b} \end{matrix} & \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix} \\
\begin{matrix} -\frac{1}{2} & 0 & 0 & \frac{m_s}{m_b} \\ 0 & 0 & -\frac{m_s}{2m_b} & \frac{m_s}{m_b} \\ 0 & \frac{m_s}{m_b} & -\frac{1}{2} & 0 \\ -\frac{m_s}{2m_b} & \frac{m_s}{m_b} & 0 & 0 \end{matrix} & \begin{matrix} -\frac{m_s}{m_b} & -\frac{m_s}{2m_b} & 0 & 0 \\ -\frac{m_s}{m_b} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{m_s}{m_b} & -\frac{m_s}{2m_b} \\ 0 & -\frac{1}{2} & -\frac{m_s}{m_b} & 0 \end{matrix} & \begin{matrix} -\frac{1}{32} & 0 \\ 0 & -\frac{m_s}{32m_b} \\ 0 & -\frac{1}{32} \\ -\frac{m_s}{32m_b} & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}
\end{bmatrix} \tag{58}$$

B Anomalous Dimension Matrix

Considering that RGE's in EFT's already represent a significantly equiped industry, we have decided to collect here our results for the anomalous dimension matrix γ_C, so that one may easily implement this matrix without having to refer to our discussion in section 2.3.2 and sequels, oriented at a more progressive, if exhaustive, solution. Note however that, due to the size of the matrix, we are bound to present it block after block. Remember that our operators are ordered as in Eq.(4,5,6) and that the chiralities follow the ordering LL, LR, RR, RL, or L, R (depending on the number of chirality indices the operator carries). The definition of the blocks in chirality space

is reminded at the very end of this section. Recalling Eq.(15):

$$\gamma_C = \begin{bmatrix} \gamma_C^{ll} & 0 & 0 \\ \gamma_C^{ql} & \gamma_C^{qq} & \gamma_C^{qg} \\ \gamma_C^{gl} & \gamma_C^{gq} & \gamma_C^{gg} \end{bmatrix} \tag{59}$$

- *Subblock* γ_C^{ll}

We remind the reader that operators with different lepton flavours do not mix (at least as long as massless neutrinos are considered) so that the subblock γ_C^{ll} is diagonal in lepton-flavour space:

$$\gamma_C^{ll} = \begin{bmatrix} \tilde{\gamma}_C^{ee} & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{\gamma}_C^{\mu\mu} & 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\gamma}_C^{\tau\tau} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{\gamma}_C^{\nu_e\nu_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{\gamma}_C^{\nu_\mu\nu_\mu} & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{\gamma}_C^{\nu_\tau\nu_\tau} \end{bmatrix} \tag{60}$$

From Eq.(16) and sequels, we read, for any lepton flavour l (but note that the basis of operators is shortened in the neutrino case, due to the absence of low-energy right-handed neutrinos):

$$\tilde{\gamma}_C^{ll} = \frac{2\alpha_S C_2(3)}{4\pi} \begin{bmatrix} -\frac{23}{4}\mathbb{1} & 0 & 0 & 0 & 0 \\ 0 & -\frac{11}{4}\mathbb{1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{7}{4} & 0 & 0 \\ 0 & 2\left(\mathbb{1} + \frac{m_s}{m_b}\Sigma\right) & 0 & -\frac{19}{4} & 0 \\ 0 & 0 & 0 & 0 & -\frac{23}{4}\mathbb{1} \end{bmatrix} \tag{61}$$

- *Subblock* γ_C^{ql}

$$\gamma_C^{ql} = \begin{bmatrix} \tilde{\gamma}_C^{ue} & \tilde{\gamma}_C^{u\mu} & \tilde{\gamma}_C^{u\tau} & \tilde{\gamma}_C^{u\nu_e} & \tilde{\gamma}_C^{u\nu_\mu} & \tilde{\gamma}_C^{u\nu_\tau} \\ \tilde{\gamma}_C^{de} & \tilde{\gamma}_C^{d\mu} & \tilde{\gamma}_C^{d\tau} & \tilde{\gamma}_C^{d\nu_e} & \tilde{\gamma}_C^{d\nu_\mu} & \tilde{\gamma}_C^{d\nu_\tau} \\ \tilde{\gamma}_C^{se} & \tilde{\gamma}_C^{s\mu} & \tilde{\gamma}_C^{s\tau} & \tilde{\gamma}_C^{s\nu_e} & \tilde{\gamma}_C^{s\nu_\mu} & \tilde{\gamma}_C^{s\nu_\tau} \\ \tilde{\gamma}_C^{ce} & \tilde{\gamma}_C^{c\mu} & \tilde{\gamma}_C^{c\tau} & \tilde{\gamma}_C^{c\nu_e} & \tilde{\gamma}_C^{c\nu_\mu} & \tilde{\gamma}_C^{c\nu_\tau} \\ \tilde{\gamma}_C^{be} & \tilde{\gamma}_C^{b\mu} & \tilde{\gamma}_C^{b\tau} & \tilde{\gamma}_C^{b\nu_e} & \tilde{\gamma}_C^{b\nu_\mu} & \tilde{\gamma}_C^{b\nu_\tau} \end{bmatrix} \tag{62}$$

From Eq.(16) and sequels, we read, for any lepton flavour l and $q = u, d, c$:

$$\tilde{\gamma}_C^{ql} = -\frac{2\alpha_S C_2(3)}{4\pi}\frac{3}{2}Q_l Q_q \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{m_b}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{3m_b}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \end{bmatrix} \tag{63}$$

For $q = b, s$, we have:

$$\tilde{\gamma}_C^{ql} = \frac{2\alpha_S C_2(3)}{4\pi}\frac{3}{2}Q_l Q_q \left\{ - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{m_b}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \end{bmatrix} + \tilde{V}^q_{\text{Fierz}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 \\ 0 & \frac{m_q}{3m_b}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \end{bmatrix} \right\} \tag{64}$$

- *Subblock* γ_C^{gl}

$$\gamma_C^{gl} = \begin{bmatrix} \tilde{\gamma}_C^{ge} & \tilde{\gamma}_C^{g\mu} & \tilde{\gamma}_C^{g\tau} & \tilde{\gamma}_C^{g\nu_e} & \tilde{\gamma}_C^{g\nu_\mu} & \tilde{\gamma}_C^{g\nu_\tau} \end{bmatrix} \tag{65}$$

From Eq.(16) and sequels, we read, for any lepton flavour l:

$$\tilde{\gamma}_C^{gl} = \frac{2\alpha_S C_2(3)}{4\pi}\frac{1}{3}Q_l \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \Xi + \frac{m_s}{m_b}\Phi & 0 & 0 & 0 \\ 0 & 2[\Xi' + \frac{m_s}{m_b}\Phi'] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{66}$$

- *Subblock* γ_C^{qq}

$$\gamma_C^{ql} = \begin{bmatrix} \tilde{\gamma}_C^{uu} & \tilde{\gamma}_C^{ud} & \tilde{\gamma}_C^{us} & \tilde{\gamma}_C^{uc} & \tilde{\gamma}_C^{ub} \\ \tilde{\gamma}_C^{du} & \tilde{\gamma}_C^{dd} & \tilde{\gamma}_C^{ds} & \tilde{\gamma}_C^{dc} & \tilde{\gamma}_C^{db} \\ \tilde{\gamma}_C^{su} & \tilde{\gamma}_C^{sd} & \tilde{\gamma}_C^{ss} & \tilde{\gamma}_C^{sc} & \tilde{\gamma}_C^{sb} \\ \tilde{\gamma}_C^{cu} & \tilde{\gamma}_C^{cd} & \tilde{\gamma}_C^{cs} & \tilde{\gamma}_C^{cc} & \tilde{\gamma}_C^{cb} \\ \tilde{\gamma}_C^{bu} & \tilde{\gamma}_C^{bd} & \tilde{\gamma}_C^{bs} & \tilde{\gamma}_C^{bc} & \tilde{\gamma}_C^{bb} \end{bmatrix} \tag{67}$$

For two quark flavours q and q', we read from Eq.(20) and sequels,
* if q and $q' \neq b, s$:

$$\tilde{\gamma}_C^{q'q} = \frac{2\alpha_S C_2(3)}{4\pi}\left\{\delta^{q'q}\mathrm{diag}(5\mathbb{1}, 5\mathbb{1}, 5\mathbb{1}, 2\mathbb{1}, 2\mathbb{1}, 5\mathbb{1}, 5\mathbb{1}, 5\mathbb{1}, 2\mathbb{1}, 2\mathbb{1}) - \delta^{q'q}\tilde{\mathcal{M}}^{q'q}_{\text{diag}} - \tilde{\mathcal{M}}^{q'q}_{\text{univ.}}\right\} \tag{68}$$

* if q and $q' = b$ or s:

$$\tilde{\gamma}_C^{q'q} = \frac{2\alpha_S C_2(3)}{4\pi}\left\{\delta^{q'q}\mathrm{diag}(5\mathbb{1}, 5\mathbb{1}, 5\mathbb{1}, 2\mathbb{1}, 2\mathbb{1}) - \delta^{q'q}[\mathbb{1}, 0]\tilde{\mathcal{M}}^{q'q}_{\text{diag}}\begin{bmatrix} \mathbb{1} \\ 0 \end{bmatrix} - [\mathbb{1}, -\tilde{V}^{q'}_{\text{Fierz}}]\tilde{\mathcal{M}}^{q'q}_{\text{univ.}}\begin{bmatrix} \mathbb{1} \\ -\tilde{V}^{q}_{\text{Fierz}} \end{bmatrix}\right\} \tag{69}$$

* if $q \neq b, s$ and $q' = b$ or s:

$$\tilde{\gamma}_C^{q'q} = -\frac{2\alpha_S C_2(3)}{4\pi}[\mathbb{1}, -\tilde{V}^{q'}_{\text{Fierz}}]\tilde{\mathcal{M}}^{q'q}_{\text{univ.}} \tag{70}$$

* if $q = b$ or s and $q' \neq b, s$:

$$\tilde{\gamma}_C^{q'q} = -\frac{2\alpha_S C_2(3)}{4\pi}\tilde{\mathcal{M}}^{q'q}_{\text{univ.}}\begin{bmatrix} \mathbb{1} \\ -\tilde{V}^{q}_{\text{Fierz}} \end{bmatrix} \tag{71}$$

with:

$$\tilde{\mathcal{M}}^{q'q}_{\text{diag}} = \begin{bmatrix} 8\mathbb{1} & 0 & \frac{1}{32}\Delta & 0 & 0 & 0 & 0 & -\frac{3}{32}\Delta & 0 & 0 \\ 0 & 2\mathbb{1} + \frac{3}{4}\Sigma_3 & 0 & 0 & 0 & 0 & -\frac{9}{4}\Sigma_3 & 0 & 0 & 0 \\ 24\Delta^T & 0 & 0 & 0 & 0 & -72\Delta^T & 0 & 0 & 0 & 0 \\ A_H & B_H & 0 & \mathbb{1} & 0 & C_H & D_H & 0 & 0 & 0 \\ A_{\tilde{H}} & B_{\tilde{H}} & 0 & 0 & \mathbb{1} & C_{\tilde{H}} & D_{\tilde{H}} & 0 & 0 & 0 \\ \frac{9}{4}\mathbb{1} & 0 & -\frac{3}{64}\Delta & 0 & 0 & \frac{5}{4}\mathbb{1} & 0 & -\frac{7}{64}\Delta & 0 & 0 \\ 0 & -\frac{9}{8}(\mathbb{1} + \Sigma_3) & 0 & 0 & 0 & 0 & \frac{1}{8}(43\mathbb{1} - 21\Sigma_3) & 0 & 0 & 0 \\ -36\Delta^T & 0 & -\frac{9}{4}\mathbb{1} & 0 & 0 & -84\Delta^T & 0 & \frac{27}{4}\mathbb{1} & 0 & 0 \\ A_{\mathcal{H}} & B_{\mathcal{H}} & 0 & 0 & 0 & C_{\mathcal{H}} & D_{\mathcal{H}} & 0 & \mathbb{1} & 0 \\ A_{\tilde{\mathcal{H}}} & B_{\tilde{\mathcal{H}}} & 0 & 0 & 0 & C_{\tilde{\mathcal{H}}} & D_{\tilde{\mathcal{H}}} & 0 & 0 & \mathbb{1} \end{bmatrix}$$

$$
\tilde{\mathcal{M}}^{q'q}_{\text{univ}} = \frac{1}{4}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 & 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 & 0 & -(\mathbb{1}+\tilde{\Sigma}) & 0 \\
0 & -\frac{m_q}{3m_b}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0 & 0 & \frac{m_q}{m_b}(\mathbb{1}+\tilde{\Sigma}) & 0 & 0 & 0
\end{bmatrix}
$$

- *Subblock* γ_C^{gq}

$$
\gamma_C^{gq} = \begin{bmatrix} \tilde{\gamma}_C^{gu} & \tilde{\gamma}_C^{gd} & \tilde{\gamma}_C^{gs} & \tilde{\gamma}_C^{gc} & \tilde{\gamma}_C^{gb} \end{bmatrix} \tag{72}
$$

For a quark flavour $q = u, d, s$, we read from Eq.(20) and sequels:

$$
\tilde{\gamma}_C^{gq} = -\frac{2\alpha_S C_2(3)}{4\pi}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-12\frac{m_q}{m_b}\Xi & \frac{1}{12}(\Xi+\frac{m_s}{m_b}\Phi) & 0 & 0 & 0 & 0 & -\frac{1}{4}(\Xi+\frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\
24\frac{m_q}{m_b}\tilde{\Xi} & \frac{1}{6}(\Xi'+\frac{m_s}{m_b}\Phi') & 0 & 0 & 0 & 0 & -\frac{1}{2}(\Xi'+\frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\
-\frac{11}{8}\frac{m_q}{m_b}\Xi & \frac{7}{144}(\Xi+\frac{m_s}{m_b}\Phi) & 0 & 0 & 0 & -\frac{15}{8}\frac{m_q}{m_b}\Xi & -\frac{7}{48}(\Xi+\frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\
\frac{11}{4}\frac{m_q}{m_b}\tilde{\Xi} & \frac{7}{72}(\Xi'+\frac{m_s}{m_b}\Phi') & 0 & 0 & 0 & \frac{15}{4}\frac{m_q}{m_b}\tilde{\Xi} & -\frac{7}{24}(\Xi'+\frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\
0 & 0 & -\frac{3}{64}\frac{m_q}{m_b}\mathbb{1} & 0 & 0 & 0 & 0 & \frac{9}{64}\frac{m_q}{m_b}\mathbb{1} & 0 & 0
\end{bmatrix} \tag{73}
$$

For $q = b, s$, we have:

$$
\tilde{\gamma}_C^{gq} = \frac{2\alpha_S C_2(3)}{4\pi}\times
$$
$$
\left\{ -\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-12\frac{m_q}{m_b}\Xi & \frac{1}{12}(\Xi+\frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\
24\frac{m_q}{m_b}\tilde{\Xi} & \frac{1}{6}(\Xi'+\frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\
-\frac{11}{8}\frac{m_q}{m_b}\Xi & \frac{7}{144}(\Xi+\frac{m_s}{m_b}\Phi) & 0 & 0 & \\
\frac{11}{4}\frac{m_q}{m_b}\tilde{\Xi} & \frac{7}{72}(\Xi'+\frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\
0 & 0 & -\frac{3}{64}\frac{m_q}{m_b}\mathbb{1} & 0 & 0
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{4}(\Xi+\frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\
0 & -\frac{1}{2}(\Xi'+\frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\
-\frac{15}{8}\frac{m_q}{m_b}\Xi & -\frac{7}{48}(\Xi+\frac{m_s}{m_b}\Phi) & 0 & 0 & 0 \\
\frac{15}{4}\frac{m_q}{m_b}\tilde{\Xi} & -\frac{7}{24}(\Xi'+\frac{m_s}{m_b}\Phi') & 0 & 0 & 0 \\
0 & 0 & \frac{9}{64}\frac{m_q}{m_b}\mathbb{1} & 0 & 0
\end{bmatrix}\tilde{V}^q_{\text{Fierz}} \right\} \tag{74}
$$

- *Subblock* γ_C^{qg}

$$
\gamma_C^{qg} = \begin{bmatrix} \tilde{\gamma}_C^{ug} \\ \tilde{\gamma}_C^{dg} \\ \tilde{\gamma}_C^{sg} \\ \tilde{\gamma}_C^{cg} \\ \tilde{\gamma}_C^{bg} \end{bmatrix} \tag{75}
$$

For a quark flavour $q = u, d, s$, we read from Eq.(23) and sequels:

$$\tilde{\gamma}_C^{qg} = -\frac{2\alpha_S C_2(3)}{4\pi} \begin{bmatrix} 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 18Q_q\frac{m_q}{m_b}\mathbb{1} & 0 & 0 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 6Q_q\frac{m_q}{m_b}\mathbb{1} & 6\frac{m_q}{m_b}\mathbb{1} & 0 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{bmatrix} \tag{76}$$

For $q = b, s$, we have:

$$\tilde{\gamma}_C^{gq} = \frac{2\alpha_S C_2(3)}{4\pi} \left\{ -\begin{bmatrix} 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 18Q_q\frac{m_q}{m_b}\mathbb{1} & 0 & 0 & \cdots \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{bmatrix} + \tilde{V}^q_{\text{Fierz}} \begin{bmatrix} 0 & 0 & 0 & \\ 0 & 0 & 0 & \\ 6Q_q\frac{m_q}{m_b}\mathbb{1} & 6\frac{m_q}{m_b}\mathbb{1} & 0 & \cdots \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{bmatrix} \right\} \tag{77}$$

- *Subblock* γ_C^{gg}

$$\gamma_C^{gg} = \frac{2\alpha_S C_2(3)}{4\pi} \left[\begin{array}{cccccccc} \frac{5}{4}\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4Q_d\mathbb{1} & \frac{3}{4}\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{35}{4}\mathbb{1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{35}{4}\mathbb{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{19}{4}\mathbb{1} & 0 & 0 & 0 \\ \frac{1}{4}\Omega & 0 & \frac{2}{3}Q_d\mathbb{1} & \frac{1}{3}Q_d\sigma_3 & 0 & -\frac{5}{24}\mathbb{1} & -\frac{11}{48}\sigma_3 & 0 \\ -\frac{1}{2}\tilde{\Omega} & 0 & \frac{4}{3}Q_d\sigma_3 & \frac{2}{3}Q_d\mathbb{1} & 0 & -\frac{11}{12}\sigma_3 & -\frac{5}{24}\mathbb{1} & 0 \\ -(1-\frac{m_s^2}{m_b^2})\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{11}{4}\mathbb{1} \\ 0 & \frac{3}{8}\Omega & 0 & 0 & 0 & Q_d\mathbb{1} & \frac{1}{2}Q_d\sigma_3 & 0 \\ 0 & -\frac{3}{4}\tilde{\Omega} & 0 & 0 & 0 & 2Q_d\sigma_3 & Q_d\mathbb{1} & 0 \\ 0 & \frac{7}{32}\Omega & 0 & 0 & 0 & \frac{7}{12}Q_d\mathbb{1} & \frac{7}{24}Q_d\sigma_3 & 0 \\ 0 & -\frac{7}{16}\tilde{\Omega} & 0 & 0 & 0 & \frac{7}{6}Q_d\sigma_3 & \frac{7}{12}Q_d\mathbb{1} & 0 \\ 0 & \frac{9}{8}(1+\frac{m_s^2}{m_b^2})\mathbb{1} & 0 & 0 & 0 & \frac{9}{2}Q_d\mathbb{1} & -\frac{9}{4}Q_d\sigma_3 & 0 \end{array} \right.$$

$$\left. \begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\frac{35}{4}\mathbb{1} & 0 & \mathbb{1} & \frac{1}{2}\sigma_3 & -\frac{9}{2}\mathbb{1} \\ 0 & -\frac{35}{4}\mathbb{1} & 2\sigma_3 & \mathbb{1} & 9\sigma_3 \\ -\frac{21}{16}\mathbb{1} & \frac{3}{32}\sigma_3 & -\frac{385}{24}\mathbb{1} & \frac{41}{48}\sigma_3 & -\frac{11}{8}\mathbb{1} \\ \frac{3}{8}\sigma_3 & -\frac{21}{16}\mathbb{1} & \frac{41}{12}\sigma_3 & -\frac{385}{24}\mathbb{1} & \frac{11}{4}\sigma_3 \\ -\frac{3}{2}\mathbb{1} & \frac{3}{4}\sigma_3 & -\frac{45}{2}\mathbb{1} & \frac{45}{4}\sigma_3 & 2\mathbb{1} \end{array} \right] \tag{78}$$

We finally collect the definition of the various relevant subblocks in chirality space (with exception of the identity $\mathbb{1}$ and the null matrix 0):

$$\Sigma \equiv \begin{matrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{matrix} \; ; \; \tilde{\Sigma} \equiv \begin{matrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{matrix} \; ; \; \begin{cases} \Xi \equiv \begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{matrix} \\ \\ \Phi \equiv \begin{matrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{matrix} \end{cases} ; \; \begin{cases} \Xi' \equiv \begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{matrix} \\ \\ \Phi' \equiv \begin{matrix} 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 \end{matrix} \end{cases} \tag{79}$$

$$\tilde{\Xi} \equiv \begin{matrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{matrix} \;;\; \sigma_1 \equiv \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \;;\; \sigma_3 \equiv \begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix} \;;\; \varepsilon \equiv \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix}$$

$$\Sigma_3 = \begin{matrix} ; \; 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{matrix} \;;\; E = \begin{matrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{matrix} \;;\; \tilde{E} = \begin{matrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{matrix} \;;\; \Delta = \begin{matrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix}$$

$$\begin{cases} A_H = \frac{m_q}{4m_b}\left[3(\mathbb{1}+\tilde{\Sigma}) + \Sigma_3 + E\right] \\ A_{\tilde{H}} = \frac{1}{4}\left[3\mathbb{1} + \Sigma_3 + \frac{m_s}{m_b}(3\Sigma + \tilde{E})\right] \\ A_{\mathcal{H}} = -\frac{3}{8}\frac{m_q}{m_b}\left[3(\mathbb{1}+\tilde{\Sigma}) + \Sigma_3 + E\right] \\ A_{\tilde{\mathcal{H}}} = -\frac{3}{8}(3\mathbb{1} + \Sigma_3) - \frac{3}{8}\frac{m_s}{m_b}(3\Sigma + \tilde{E}) \end{cases} ; \quad \begin{cases} B_H = \frac{1}{4}\left[-8\mathbb{1} + \Sigma_3 + \frac{m_s}{m_b}(-8\Sigma + \tilde{E})\right] \\ B_{\tilde{H}} = \frac{m_q}{4m_b}\left[-8(\mathbb{1}+\tilde{\Sigma}) + \Sigma_3 + E\right] \\ B_{\mathcal{H}} = -\frac{3}{8}(3\mathbb{1} + \Sigma_3) - \frac{3}{8}\frac{m_s}{m_b}(3\Sigma + \tilde{E}) \\ B_{\tilde{\mathcal{H}}} = -\frac{3}{8}\frac{m_q}{m_b}\left[3(\mathbb{1}+\tilde{\Sigma}) + \Sigma_3 + E\right] \end{cases}$$

$$\begin{cases} C_H = -\frac{3}{4}\frac{m_q}{m_b}\left[3(\mathbb{1}+\tilde{\Sigma}) + \Sigma_3 + E\right] \\ C_{\tilde{H}} = -\frac{3}{4}\left[(3\mathbb{1} + \Sigma_3) + \frac{m_s}{m_b}(3\Sigma + \tilde{E})\right] \\ C_{\mathcal{H}} = -\frac{7}{8}\frac{m_q}{m_b}\left[3(\mathbb{1}+\tilde{\Sigma}) + \Sigma_3 + E\right] \\ C_{\tilde{\mathcal{H}}} = -\frac{7}{8}(3\mathbb{1} + \Sigma_3) - \frac{7}{8}\frac{m_s}{m_b}(3\Sigma + \tilde{E}) \end{cases} ; \quad \begin{cases} D_H = -\frac{3}{4}(\Sigma_3 + \frac{m_s}{m_b}\tilde{E}) \\ D_{\tilde{H}} = -\frac{3}{4}\frac{m_q}{m_b}(\Sigma_3 + E) \\ D_{\mathcal{H}} = \frac{1}{8}(11\mathbb{1} - 7\Sigma_3) + \frac{m_s}{8m_b}(11\Sigma - 7\tilde{E}) \\ D_{\tilde{\mathcal{H}}} = \frac{m_q}{8m_b}\left[11(\mathbb{1}+\tilde{\Sigma}) - 7(\Sigma_3 + E)\right] \end{cases}$$

$$\Omega \equiv (1 + \frac{m_s^2}{m_b^2})\mathbb{1} - \frac{2}{3}\frac{m_s}{m_b}\sigma_1 \;;\; \tilde{\Omega} \equiv (1 + \frac{m_s^2}{m_b^2})\sigma_3 - \frac{2}{3}\frac{m_s}{m_b}\varepsilon$$

C Diagonalization of the anomalous-dimension matrix for dimension 7 operators

C.1 Gluonic operators

We consider the matrix $\mathcal{A}_{\mathcal{Q}\mathcal{Q}}$ defined in Eq.(31):

$$\mathcal{A}_{\mathcal{Q}\mathcal{Q}} = \begin{bmatrix} \frac{35}{4}\mathbb{1} & 0 & -\mathbb{1} & -\frac{1}{2}\sigma_3 & \frac{9}{2}\mathbb{1} \\ 0 & \frac{35}{4}\mathbb{1} & -2\sigma_3 & -\mathbb{1} & -9\sigma_3 \\ \frac{21}{16}\mathbb{1} & -\frac{3}{32}\sigma_3 & \frac{385}{24}\mathbb{1} & -\frac{41}{48}\sigma_3 & \frac{11}{8}\mathbb{1} \\ -\frac{3}{8}\sigma_3 & \frac{21}{16}\mathbb{1} & -\frac{41}{12}\sigma_3 & \frac{385}{24}\mathbb{1} & -\frac{11}{4}\sigma_3 \\ \frac{3}{2}\mathbb{1} & -\frac{3}{4}\sigma_3 & \frac{45}{2}\mathbb{1} & -\frac{45}{4}\sigma_3 & -2\mathbb{1} \end{bmatrix} = \mathcal{O}_\mathcal{Q}\mathcal{D}_\mathcal{Q}\mathcal{O}_\mathcal{Q}^{-1} \tag{80}$$

Its diagonalization happens to be accidentally tractable:

$$\mathcal{D}_\mathcal{Q} = \mathrm{diag}\left[-\frac{19}{4}\mathbb{1}, \frac{1}{8}(117 - \sqrt{3073})\mathbb{1}, \frac{1}{24}(277 - \sqrt{3193})\mathbb{1}, \frac{1}{24}(277 + \sqrt{3193})\mathbb{1}, \frac{1}{8}(117 + \sqrt{3073})\mathbb{1}\right]$$

$$\mathcal{O}_\mathcal{Q} = \begin{bmatrix} 1 & -\frac{1}{12}(47 + \sqrt{3073}) & -\frac{1}{54}(67 + \sqrt{3193} & \frac{1}{2} & \frac{1}{24}(-47 + \sqrt{3073}) \\ -2 & \frac{1}{6}(47 + \sqrt{3073}) & -\frac{1}{27}(67 + \sqrt{3193}) & 1 & \frac{1}{12}(47 - \sqrt{3073}) \\ \frac{7}{60} & 1 & \frac{1}{2} & -\frac{1}{96}(67 + \sqrt{3193}) & \frac{1}{2} \\ -\frac{7}{30} & -2 & 1 & -\frac{1}{48}(67 + \sqrt{3193}) & -1 \\ -3 & 2 & 0 & 0 & 1 \end{bmatrix} \tag{81}$$

so that the exponentiation is fully determined.

C.2 Hybrid $(\bar{b}s)$-Gauge operators

We consider the matrix $\mathcal{A}_{\mathcal{H}\mathcal{H}}$ defined in Eq.(32):

$$\mathcal{A}_{\mathcal{H}\mathcal{H}} = \begin{bmatrix} \frac{5}{24}\mathbb{1} & \frac{11}{48}\sigma_3 & 0 \\ \frac{11}{12}\sigma_3 & \frac{5}{24}\mathbb{1} & 0 \\ 0 & 0 & \frac{11}{4}\mathbb{1} \end{bmatrix} = \mathcal{O}_\mathcal{H}\mathcal{D}_\mathcal{H}\mathcal{O}_\mathcal{H}^{-1} \tag{82}$$

Its diagonalization is straigtforward:

$$\mathcal{O}_{\mathcal{H}} \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbb{1} & \frac{1}{2}\sigma_3 & 0 \\ -2\sigma_3 & \mathbb{1} & 0 \\ 0 & 0 & \sqrt{2}\mathbb{1} \end{bmatrix} \quad ; \quad \mathcal{D}_{\mathcal{H}} \equiv \mathrm{diag}\left[-\frac{1}{4}\mathbb{1}, \frac{2}{3}\mathbb{1}, \frac{11}{4}\mathbb{1}\right] \tag{83}$$

allowing for a simple exponentiation:

$$\exp\left[\frac{C_2(3)}{\beta_0}\mathcal{A}_{\mathcal{H}\mathcal{H}} \ln\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right] = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{5C_2(3)}{24\beta_0}} \begin{bmatrix} \cosh\vartheta(\mu)\mathbb{1} & \frac{1}{2}\sinh\vartheta(\mu)\sigma_3 & 0 \\ -2\sinh\vartheta(\mu)\sigma_3 & \cosh\vartheta(\mu)\mathbb{1} & 0 \\ 0 & 0 & \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\frac{61C_2(3)}{24\beta_0}}\mathbb{1} \end{bmatrix}$$
$$; \quad \vartheta(\mu) \equiv \frac{11C_2(3)}{24\beta_0}\ln\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)} \tag{84}$$

C.3 $(\bar{b}s)(\bar{q}q)$ sector

We consider the matrix $\mathcal{A}_{qq}$ defined in Eq.(36):

$$\mathcal{A}_{qq} = \frac{1}{4}[\mathbb{1}; -\tilde{V}^{q'}_{\text{Fierz}}]_{q'=b,s} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{3}(\mathbb{1}+\tilde{\Sigma}) & 0 & \mathbb{1}+\tilde{\Sigma} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbb{1} \\ -\tilde{V}^{q}_{\text{Fierz}} \end{bmatrix}_{q=b,s}$$
$$= \mathcal{O}_q \mathrm{diag}(0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{8}{3}\mathcal{D}_{24})\mathcal{O}_q^{-1} \tag{85}$$

Let us introduce the following blocks:

$$\mathcal{D}_{24} \equiv \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \quad ; \quad \mathcal{R} \equiv \frac{1}{\sqrt{2}} \begin{matrix} -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{matrix} \; ; \tag{86}$$

$$\mathcal{U}^s \equiv \frac{1}{\sqrt{2}} \begin{matrix} 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{matrix} \; ; \; \mathcal{U}^b \equiv \frac{1}{\sqrt{2}} \begin{matrix} -1 & 0 & 0 & 0 & 0 & -4 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -4 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{matrix}$$

$$
\mathcal{O}_q = \left[\begin{array}{cccccccccccccccc}
\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\mathbb{1}}{3} & 0 & \mathcal{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\mathbb{1}}{3} & 0 & \mathcal{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mathbb{1}}{3} & 0 & \mathcal{R} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\mathcal{U}^s$}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\mathcal{U}^b$}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & &
\end{array}\right]
$$

$$
\times \left[\begin{array}{cccccccccccccccc}
\mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{D}_{24} & 0 & \mathcal{D}_{24} & 0 \\
0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\mathcal{D}_{24} & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & \mathcal{D}_{24} & 0 & \mathcal{D}_{24} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\mathcal{D}_{24} & 0 & 0 & 0 & \mathbb{1}-\mathcal{D}_{24} & 0 & \mathcal{D}_{24} & 0 & \mathcal{D}_{24} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{D}_{24} & 0 & \mathbb{1}-\frac{16}{7}\mathcal{D}_{24} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{1} & 0 & \mathcal{D}_{24} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathcal{D}_{24} & 0 & -\frac{9}{7}\mathcal{D}_{24} & 0 & \mathbb{1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{1}
\end{array}\right] \tag{87}
$$

$\mathcal{O}_q$ turns out to diagonalize $\mathcal{A}_{qq}$ satisfactorily.

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